Fall 1998
Math 151
Common Exam 3
Test Form A

PRINT: Last Name: _____ **First Name:** _____

Signature: _____ **ID:** _____

Instructor's Name: _____ **Section #** _____

INSTRUCTIONS

1. In **Part 1** (Problems 1–13), mark the correct choice on your ScanTron form using a #2 pencil. *For your own records, also record your choices on your exam!* The ScanTrons will be collected after 1 hour; they will NOT be returned.
2. In **Part 2** (Problems 14–18), write all solutions in the space provided. You may use the back of any page for scratch work, but all work to be graded must be shown in the space provided. CLEARLY INDICATE YOUR FINAL ANSWERS.

PART 1: MULTIPLE-CHOICE PROBLEMS

Each problem is worth 4 points; NO partial credit will be given. Calculators may NOT be used on this part. ScanTron forms will be collected after 1 hour.

1. Evaluate $\lim_{x\to 1}\left(\dfrac{1}{x-1}-\dfrac{2}{x^2-1}\right)$.

   (a) 0 (b) $1/2$ (c) ∞ (d) 1 (e) 2

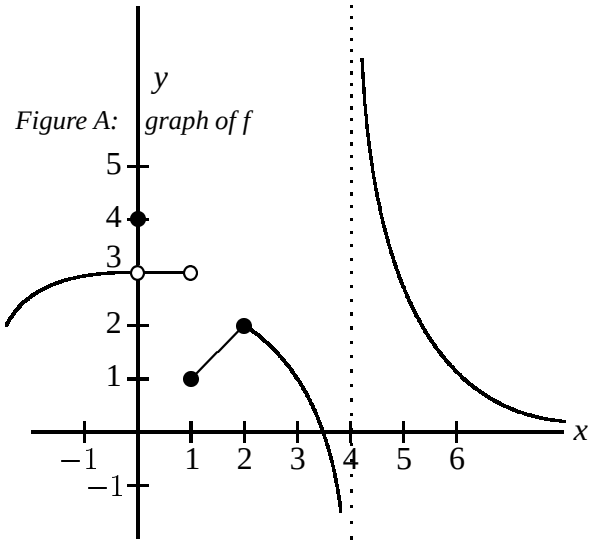
2. $\lim_{x\to 2}\dfrac{\sqrt{4x+1}-3}{x-2}=$.

   (a) 1 (b) $\sqrt{2}/6$ (c) $2/3$ (d) 0 (e) ∞

3. Find the horizontal asymptote of $f(x)=\dfrac{x-\sqrt{x^2+4x-1}}{2}$.

   (a) $y=0$ (b) $y=-1$ (c) There is none. (d) $x=1/2$ (e) $y=1/2$

Figure A: graph of f



Problems 4 and 5 refer to the graph at right.

4. Refer to the graph of f in *Figure A*. Which one of the following statements is true?

   (a) The function f is continuous at $x = 0$.

   (b) The function f is discontinuous at $x = 2$.

   (c) The function f is continuous from the left at $x = 1$.

   (d) The function f is continuous from the right at $x = 1$.

   (e) The function f is continuous at $x = 4$.

5. Refer to the graph of f in *Figure A*. Which one of the following statements is true?

   (a) The function f is differentiable at $x = 0$.

   (b) The function f is differentiable at $x = 1$.

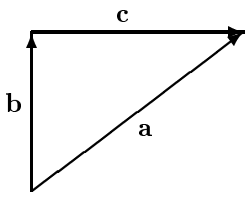
   (c) The function f is differentiable at $x = 2$.

   (d) The function f is differentiable at $x = 4$.

   (e) None of the four preceding statements is true.

6. For what t does $\langle 5, -3\rangle + t\langle -2, 1\rangle$ have direction *opposite* $\langle -1, 1\rangle$?
   (a) -1 (b) 8 (c) -2 (d) 3 (e) none of these

7. Refer to diagram below. Which of the following is equal to the vector $\mathbf{c}$?



(a) $\text{comp}_{\mathbf{a}}\mathbf{b}$ (b) $\mathbf{a}+\mathbf{b}$ (c) $\mathbf{a}\cdot\mathbf{b}$ (d) $\mathbf{a}-\mathbf{b}$ (e) $\mathbf{b}-\mathbf{a}$

8. Find the derivative of $f(x)=\dfrac{x^5}{x^3-2}$.

(a) x^2 (b) $\dfrac{5x^4}{3x^2}$ (c) $\dfrac{\frac{1}{6}x^6}{\frac{1}{4}x^4-2x}$ (d) $x^2+\dfrac{2x^2}{x^3-2}$ (e) $\dfrac{2x^7-10x^4}{(x^3-2)^2}$

9. Find an equation of the tangent line to the curve $f(x)=x^3-x$ at $x=1$.

(a) $y=0$ (b) $3x-y=3$ (c) $2x+y=2$ (d) $2x-y=1$ (e) $2x-y=2$

10. Find the distance from the point $(-2,\ 3)$ to the line $x = 4t,\ \ y = 3t + \dfrac{5}{4}$.

(a) $2\sqrt{5}$ (b) $13/5$ (c) 3 (d) $22/5$ (e) $\sqrt{6}$

11. The difference of two *unit* vectors has length 1. What is the length of their sum?

(a) $\sqrt{2}$ (b) 2 (c) $\sqrt{3}$ (d) 1 (e) $2\sqrt{2} - 1$

PART 2: WORK-OUT PROBLEMS

Each problem is worth 8 points; partial credit is possible. Calculators are permitted ONLY AFTER the ScanTrons are collected. **SHOW ALL WORK.** *If you use a calculator, explain how.*

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12. Show that there is a number x for which $x^4 + x^3 + x - 1 = 0$.
You MUST justify your answer by citing the relevant conditions and theorem.

13. In aviation, directions ("bearings") are measured in degrees clockwise from north. Consider the following velocities.

    - The velocity of our plane is $\mathbf{v}_P = 900$ km/hr @ 30° (east of north).
    - The wind velocity is $\mathbf{v}_W = 100$ km/hr @ 90° (due east).

    Find the resultant velocity with respect to the ground $\mathbf{v}_G = \mathbf{v}_P + \mathbf{v}_W$; state its magnitude and direction (bearing).

14. Find the derivative of $f(x) = 3x^2 - x + 7$ at $x = 1$ *using the definition of derivative.*

15. The path of a rocket is $\mathbf{r}(t) = \langle\, 3000t,\ 100t\,(20 - t)\,\rangle, \quad 0 \le t \le 20$. Compute:

    - the *velocity* of the rocket when $t = 8$
    - the *speed* of the rocket when $t = 15$

16. For which values of x is the dot product $\mathbf{a} \cdot \mathbf{b}$ negative? Here $\mathbf{a} = \langle x,\ 2 \rangle$ and $\mathbf{b} = \langle 2x,\ -4 \rangle$.

17. Find a value of c such that the function

$$f(x) = \begin{cases} x^2 - c & , \quad x < 2 \\ 3c & , \quad x = 2 \\ x + c & , \quad x > 2 \end{cases}$$

is continuous at $x = 2$. *Show* that your choice of c does indeed make f continuous at $x = 2$.

18. If a potato (spud) is fired with an initial speed of 25 m/s from a spud gun vertically upward from the roof of a building 200 m high, its height h in meters is given by the following function of time t (measured in seconds).

$$h(t) = 200 + 25t - 4.9t^2$$

Answer these questions. Please include the proper units in your answers.

(a) *When* does the spud hit the ground? (Assume it just misses the roof of the building on its way down.)

(b) What is the *speed* of the potato at impact? i.e, when it hits the ground?

(c) What is the *total distance* traveled by the spud?