MATH 151

COMMON EXAM #1

FALL 2001

PART 1 ANSWERS: 1.B , 2.C , 3.E , 4.D , 5.D , 6.E , 7.C , 8.B , 9.A , 10.D , 11.A

1. If $g(x) = 2 + 3x + 4x^2 + x^3$, then $g^{'}(x) = 3 + 8x + 3x^2$.

2. If $\lim_{x\to a} f(x) = 9$ and $\lim_{x\to a} g(x) = 2$, then

$$\lim_{x\to a}\left[4\sqrt{f(x)} + 5g(x)\right] = 4\sqrt{\lim_{x\to a} f(x)} + 5\lim_{x\to a} g(x) = 4\sqrt{9} + 5(2) = 22 .$$

3. $$\lim_{x\to\infty}\frac{3x^3+2x^2+5}{2x^3+7x} = \lim_{x\to\infty}\frac{3x^3+2x^2+5}{2x^3+7x}\cdot\frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to\infty}\frac{3+\frac{2}{x}+\frac{5}{x^3}}{2+\frac{7}{x^2}} = \frac{3}{2} .$$

4. If $f(x) = x^4 + 2x + 1$, then $f^{'}(x) = 4x^3 + 2$ and $f^{'}(1) = 6$. The tangent line to the curve $y = f(x) = x^4 + 2x + 1$ at $x = 1$ passes through $(1, 4)$ and has slope $m = 6$. The equation of this line is $y - 4 = 6(x - 1)$.

5. Now $t = (1/3)(x - 2)$ and $t = (1/12)(y - 4)$, so $(1/3)(x - 2) = (1/12)(y - 4)$. So $y = 4x - 4$, which is a line with slope$= 4$.

6. If $f(x) = \begin{cases} 3x-1 & \text{if } x \le 2 \\ x^2+2 & \text{if } x > 2 \end{cases}$, then $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(x^2+2) = \left(\lim_{x\to 2^+} x\right)^2 + 2 = 6$.

7. If $s = \frac{t^4}{4} - 8t + 3$, then $v(t) = t^3 - 8$ and $v(t) = 0$ when $t^3 - 8 = 0$. Thus, $t = 2$.

8. If $h(x) = f(x)g(x)$, then $h^{'}(x) = f(x)g^{'}(x) + g(x)f^{'}(x)$ and

$$h^{'}(3) = f(3)g^{'}(3) + g(3)f^{'}(3) = (4)(5) + (2)(-6) = 8 .$$

9. Now $f(x) = 2x^3 + x^2 + 2$ is continuous on $(-\infty, \infty)$ and $f(-2) = -10 < 0$ and $f(-1) = 1 > 0$. By the Intermediate Value Theorem, there exists $x^* \in (-2, -1)$ such that $f(x^*) = 2(x^*)^3 + (x^*)^2 + 2 = 0$.

10. $$\lim_{t\to 2}\frac{2t^2+6t}{5t-5} = \frac{2\left(\lim_{t\to 2} t^2\right) + 6\left(\lim_{t\to 2} t\right)}{5\left(\lim_{t\to 2} t\right) - 5} = \frac{(2)(4)+(6)(2)}{5(2)-5} = 4 .$$

11. The vectors $\langle 1, x\rangle$ and $\langle 3-4x, 5\rangle$ are orthogonal if

$$\begin{aligned} \langle 1, x\rangle \cdot \langle 3-4x, 5\rangle &= 0 \\ (1)(3-4x) + (x)(5) &= 0 \\ 3 + x &= 0 \\ x &= -3 . \end{aligned}$$

12. If $f(x) = 1/x$, then

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\dfrac{1}{x+h} - \dfrac{1}{x}}{h} = \lim_{h\to 0} \frac{x-(x+h)}{x(x+h)h}$$
$$= \lim_{h\to 0} \frac{-h}{x(x+h)h} = \lim_{h\to 0} \frac{-1}{x(x+h)} = -\frac{1}{x^2}$$

13. Let $\mathbf{b} = 5\mathbf{i} + 12\mathbf{j}$ and $\mathbf{a} = 3\mathbf{i} - 4\mathbf{j}$. Then

$$\text{proj}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{\|\mathbf{a}\|}\left(\frac{\mathbf{a}}{\|\mathbf{a}\|}\right) = \frac{(5)(3)+(12)(-4)}{\sqrt{(3)^2+(-4)^2}}\left(\frac{3\mathbf{i}-4\mathbf{j}}{\sqrt{(3)^2+(-4)^2}}\right) = -\frac{99}{25}\mathbf{i} + \frac{132}{25}\mathbf{j}\ .$$

14. If $f(x) = \dfrac{x^2-1}{2x^2+1}$, then

$$f'(x) = \frac{(2x^2+1)(x^2-1)' - (x^2-1)(2x^2+1)'}{(2x^2+1)^2} = \frac{(2x^2+1)(2x) - (x^2-1)(4x)}{(2x^2+1)^2} = \frac{6x}{(2x^2+1)^2}\ .$$

15. If $g(x) = (1+x+2x^2)(2+x^2+x^3)$, then

$$g'(x) = (1+x+2x^2)(2+x^2+x^3)' + (2+x^2+x^3)(1+x+2x^2)' = (1+x+2x^2)(2x+3x^2) + (2+x^2+x^3)(1+4x)\ .$$

16.

$$\lim_{x\to-\infty}(x+\sqrt{x^2+6x}) = \lim_{x\to-\infty}(x+\sqrt{x^2+6x})\cdot\left(\frac{x-\sqrt{x^2+6x}}{x-\sqrt{x^2+6x}}\right)$$
$$= \lim_{x\to-\infty}\frac{x^2-(x^2+6x)}{x-\sqrt{x^2+6x}}$$
$$= \lim_{x\to-\infty}\frac{-6x}{x-\sqrt{x^2(1+(6/x))}}$$
$$= \lim_{x\to-\infty}\frac{-6x}{x-|x|\sqrt{1+(6/x)}}$$
$$= \lim_{x\to-\infty}\frac{-6x}{x+x\sqrt{1+(6/x)}}$$
$$= \lim_{x\to-\infty}\frac{-6}{1+\sqrt{1+(6/x)}} = -3$$

17. The only possible value where $f(x)$ could fail to be continuous is at $x = 2$. Now $\lim_{x\to 2} f(x)$ exists if

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$$
$$\lim_{x\to 2^-}(cx+1) = \lim_{x\to 2^+}\left(\frac{1}{4}cx^2 - 2\right)$$
$$2c+1 = c-2$$
$$c = -3\ .$$

Thus, if $c = -3$, then $f(2) = (-3)(2) + 1 = -5$ and $\lim_{x \to 2} f(x) = -5 = f(2)$. Thus, $f(x)$ is continuous at $x = 2$.

18. Now $\overrightarrow{\mathrm{RP}} = \langle -3, -2 \rangle$ and $\overrightarrow{\mathrm{RQ}} = \langle -1, -2 \rangle$, so $\|\overrightarrow{\mathrm{RP}}\| = \sqrt{13}$ and $\|\overrightarrow{\mathrm{RQ}}\| = \sqrt{5}$. Let θ be the angle at vertex R. Then

$$\begin{aligned} \overrightarrow{\mathrm{RP}} \cdot \overrightarrow{\mathrm{RQ}} &= \|\overrightarrow{\mathrm{RP}}\| \|\overrightarrow{\mathrm{RQ}}\| \cos\theta \\ (-3)(-1) + (-2)(-2) &= \sqrt{13}\sqrt{5} \cos\theta \\ \cos\theta &= \frac{7}{\sqrt{65}} \,. \end{aligned}$$