PART 1: MULTIPLE-CHOICE PROBLEMS

Each problem is worth 4 points: NO partial credit will be given. Calculators may NOT be used on this part. ScanTron forms will be collected after 1 hour.

1. If $g(x) = \sin 2x$, then $g^{(5)}(x) =$

   (a) $16 \cos 2x$

   (b) $32 \cos 2x$

   (c) $32 \sin 2x$

   (d) $-32 \cos 2x$

   (e) $\cos 32x$

2. The function $f(x) = 3x + \sin x$ is one-to-one. If g is the inverse function of f , then $g'(3\pi) =$

   (a) $\frac{1}{2\pi}$

   (b) $\frac{1}{\pi}$

   (c) 1

   (d) $\frac{1}{2}$

   (e) Can't be determined from the information given.

3. Simplify $\dfrac{e^{-x}\left(e^{2x} + 4e^{4x}\right)}{\left(e^{2x}\right)^2}$

   (a) $e^{-3x} + 4$

   (b) $e^{5x} + 4e^{7x}$

   (c) $e^{-3x} + e^{3x}$

   (d) $4e^{3x} + e^{x}$

   (e) $e^{-3x} + 4e^{-x}$

4. $\lim_{\theta \to 0} \dfrac{\sin 9\theta}{4\theta} =$

(a) $\dfrac{9}{4}$

(b) $\dfrac{4}{9}$

(c) ∞

(d) 1

(e) 0

5. If $f(x) = 2\sqrt{x + \dfrac{1}{x}}$, then $f'(x) =$

(a) $\dfrac{1 + x^{-2}}{\left(x + x^{-1}\right)^{1/2}}$

(b) $\left(1 - x^{-2}\right)^{1/2}$

(c) $\dfrac{1 - x^{-2}}{\left(x + x^{-1}\right)^{1/2}}$

(d) $\dfrac{1 + x^{2}}{\left(x + x^{-1}\right)^{1/2}}$

(e) $\dfrac{1 - x^{-2}}{\left(x - x^{-1}\right)^{1/2}}$

6. If $xy = 8$ and $dx/dt = -2$, find dy/dt when $x = 4$.

(a) -1

(b) 1

(c) 0

(d) 2

(e) -2

7. Find the derivative of $f(x) = \tan^2(x^3 + x)$

(a) $2(3x^2 + 1)\tan(x^3 + x)\sec^2(x^3 + x)$

(b) $\sec^2(x^3 + x)$

(c) $2\tan(x^3 + x)\sec(x^3 + x)$

(d) $\tan^2(3x^2 + 1)$

(e) $2(3x^2 + 1)\tan^2(x^3 + x)\sec^2(x^3 + x)$

8. $\lim_{x\to\infty} \dfrac{2e^{4x} - 7e^{-x}}{e^{4x} + 1000e^{x} + 10} =$

(a) 1

(b) $\frac{1}{2}$

(c) 2

(d) -1

(e) 0

9. The solution of $\ln(x + 4) - \ln x = 2\ln 2$ is

(a) $\frac{4}{3}$

(b) 1

(c) $\frac{2}{3}$

(d) $\frac{3}{2}$

(e) There is no solution.

10. The inverse function of $f(x) = \dfrac{2+5x}{3x+7}$ is

(a) $f^{-1}(x) = \dfrac{7x-2}{5-3x}$

(b) $f^{-1}(x) = \dfrac{3x-7}{2-5x}$

(c) $f^{-1}(x) = x$

(d) $f^{-1}(x) = \dfrac{3x+7}{2+5x}$

(e) $f^{-1}(x) = \dfrac{7x+2}{5+3x}$

11. A spherical snowball is melting in such a way that its volume is decreasing at a rate of $2 \text{ cm}^3/\text{min}$. At what rate is the radius changing when the radius is 7 cm?

(a) $-\dfrac{1}{7\pi}$ cm/min

(b) $-\dfrac{1}{49\pi}$ cm/min

(c) $\dfrac{1}{49\pi}$ cm/min

(d) $-\dfrac{1}{196\pi}$ cm/min

(e) $-\dfrac{1}{98\pi}$ cm/min

PART 2: WORK-OUT PROBLEMS

Each problem is worth 8 points. Detailed analytic solutions must be provided. Partial credit is possible. Calculators are permitted ONLY AFTER the ScanTrons are collected.

12. Consider the function $g(x) = \sqrt[3]{1+x}$.

    (a) Find the linear approximation of $g(x)$ for values of x near $a = 7$. (4 pts)

    (b) Use your answer above to approximate $\sqrt[3]{8.1}$ (4 pts)

13. Find the slope of the tangent line to the curve with the equation $2x^3 - x^2y + y^3 - 1 = 0$ at the point $(2, -3)$.

14. Find the equation of the tangent line to the curve given by the parametric equations

$$x = t\cos t$$
$$y = t\sin t$$

at the point on the curve where $t = \pi/2$.

15. Find all values of the constant r for which the function $y = e^{rx}$ satisfies the differential equation $y'' - y' - 2y = 0$.

16. Two cars are on roads that intersect at right angles, each car moving away from the intersection. At what rate is the distance between them increasing if car A is 4 miles from the intersection and going east at 60 miles per hour, while car B is 3 miles from the intersection and going north at 80 miles per hour?

17. Differentiate the following functions. Include **at least** one intermediate step.

(a) $f(x) = xe^{-(x^2/4)}$ (4 pts)

(b) $f(x) = x\cos(1/x^2)$. (4 pts)

18. The vector function $\mathbf{r}(t) = \langle t^2 \, , \, 16t - 4t^2 \rangle$ gives the position of a particle at time t. Find the time t when the velocity and acceleration of the particle are orthogonal.