MATH 151
EXAM #2B SOLUTIONS
FALL 2001

PART 1

1. The correct answer is (b).

   If $g(x) = \sin 2x$, then

$$\begin{aligned} g'(x) &= 2\cos 2x \\ g''(x) &= -4\sin 2x \\ g'''(x) &= -8\cos 2x \\ g^{(4)}(x) &= 16\sin 2x \\ g^{(5)}(x) &= 32\cos 2x \end{aligned}$$

2. The correct answer is (d).

   If $f(x) = 3x + \sin x$, then $f'(x) = 3 + \cos x$. Then

$$\begin{aligned} g'(f(x)) &= \frac{1}{f'(x)} \\ g'(f(\pi)) &= \frac{1}{f'(\pi)} \\ g'(3\pi) &= \frac{1}{2} \end{aligned}$$

3. The correct answer is (e).

$$\frac{e^{-x}\left(e^{2x}+4e^{4x}\right)}{\left(e^{2x}\right)^2} = \frac{e^{-x}\left(e^{2x}+4e^{4x}\right)}{e^{4x}} = e^{-5x}\left(e^{2x}+4e^{4x}\right) = e^{-3x}+4e^{-x}$$

4. The correct answer is (a).

$$\lim_{\theta\to 0}\frac{\sin 9\theta}{4\theta} = \lim_{\theta\to 0}\left(\frac{\sin 9\theta}{9\theta}\cdot\frac{9}{4}\right) = \frac{9}{4}\left(\lim_{\theta\to 0}\frac{\sin 9\theta}{9\theta}\right) = \frac{9}{4}\left(\lim_{t\to 0}\frac{\sin t}{t}\right) = \frac{9}{4}$$

5. The correct answer is (c).

   Now $f(x) = 2\sqrt{x+\dfrac{1}{x}} = 2\left(x+x^{-1}\right)^{1/2}$ and by the chain rule

$$f'(x) = 2\cdot\frac{1}{2}\left(x+x^{-1}\right)^{-1/2}\left(x+x^{-1}\right)' = \left(x+x^{-1}\right)^{-1/2}\left(1-x^{-2}\right) = \frac{1-x^{-2}}{\left(x+x^{-1}\right)^{1/2}}$$

6. The correct answer is (b).

   If $xy = 8$ and $x = 4$, then $y = 2$. Now

$$\begin{aligned} \frac{d}{dt}(xy) &= \frac{d}{dt}(8) \\ x\frac{dy}{dt} + y\frac{dx}{dt} &= 0 \\ 4\frac{dy}{dt} + 2(-2) &= 0 \\ \frac{dy}{dt} &= 1 \end{aligned}$$

7. The correct answer is (a).

   Now $f(x) = \tan^2(x^3 + x) = \left(\tan(x^3 + x)\right)^2$ and by the chain rule

$$\begin{aligned} f'(x) &= 2\tan(x^3 + x)\left(\tan(x^3 + x)\right)' = 2\tan(x^3 + x)\sec^2(x^3 + x)\left(x^3 + x\right)' \\ &= 2\tan(x^3 + x)\sec^2(x^3 + x)(3x^2 + 1) \end{aligned}$$

8. The correct answer is (c).

$$\begin{aligned} \lim_{x\to\infty} \frac{2e^{4x} - 7e^{-x}}{e^{4x} + 1000e^{x} + 10} &= \lim_{x\to\infty} \left(\frac{2e^{4x} - 7e^{-x}}{e^{4x} + 1000e^{x} + 10} \cdot \frac{e^{-4x}}{e^{-4x}} \right) \\ &= \lim_{x\to\infty} \frac{2 - 7e^{-5x}}{1 + 1000e^{-3x} + 10e^{-4x}} \\ &= \frac{2 - 7\left(\lim_{x\to\infty} e^{-5x}\right)}{1 + 1000\left(\lim_{x\to\infty} e^{-3x}\right) + 10\left(\lim_{x\to\infty} e^{-4x}\right)} \\ &= 2 \end{aligned}$$

9. The correct answer is (a).

$$\begin{aligned} \ln(x + 4) - \ln x &= 2\ln 2 \\ \ln\left(\frac{x + 4}{x}\right) &= \ln 2^2 = \ln 4 \\ \frac{x + 4}{x} &= 4 \\ x &= \frac{4}{3} \end{aligned}$$

10. The correct answer is (a).

To find the inverse function of $f(x) = \dfrac{2+5x}{3x+7}$ solve for x in

$$\begin{aligned} y &= \frac{2+5x}{3x+7} \\ (3x+7)y &= 2+5x \\ 3xy+7y &= 2+5x \\ 3xy-5x &= 2-7y \\ x(3y-5) &= 2-7y \\ x &= \frac{7y-2}{5-3y} \\ f^{-1}(y) &= \frac{7y-2}{5-3y} \end{aligned}$$

11. The correct answer is (e).

The volume is $V = (4/3)\pi r^3$. Since $dV/dt = -2$ cm^3/min,

$$\begin{aligned} \frac{dV}{dt} &= 4\pi r^2 \frac{dr}{dt} \\ -2 &= 4\pi (7)^2 \frac{dr}{dt} \\ \frac{dr}{dt} &= -\frac{1}{98\pi} \text{ cm/min} \end{aligned}$$

PART 2

12. If $g(x) = \sqrt[3]{1+x}$, then $g'(x) = \dfrac{1}{3(1+x)^{2/3}}$.

(a) The linear approximation of $g(x)$ for values of x near $a = 7$ is

$$L(x) = g(7) + g'(7)(x-7) = 2 + \frac{1}{12}(x-7)$$

(b) For values of x near $a = 7$

$$\begin{aligned} \sqrt[3]{1+x} &\approx 2 + \frac{1}{12}(x-7) \\ \sqrt[3]{1+7.1} &\approx 2 + \frac{1}{12}(7.1-7) \\ \sqrt[3]{8.1} &\approx 2 + \frac{1}{12}(0.1) \approx 2.0083 \end{aligned}$$

13. Find y' by implicit differentiation.

$$\begin{aligned} 2x^3 - x^2y + y^3 - 1 &= 0 \\ 6x^2 - x^2y' - 2xy + 3y^2y' &= 0 \\ 6(2)^2 - (2)^2y' - 2(2)(-3) + 3(-3)^2y' &= 0 \\ 36 + 23y' &= 0 \\ y' &= -\frac{36}{23} \end{aligned}$$

14. Now

$$\begin{aligned}\frac{dx}{dt} &= -t\sin t + \cos t \\ \frac{dy}{dt} &= t\cos t + \sin t\ ,\end{aligned}$$

so

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy/dt}{dx/dt} = \frac{t\cos t + \sin t}{-t\sin t + \cos t} \\ \left.\frac{dy}{dx}\right|_{t=\pi/2} &= \frac{\frac{\pi}{2}\cos\frac{\pi}{2} + \sin\frac{\pi}{2}}{-\frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2}} = -\frac{2}{\pi}\end{aligned}$$

Now $x(\pi/2) = 0$ and $y(\pi/2) = \pi/2$. The tangent line at the point on the curve where $t = \pi/2$ is $y - \pi/2 = -(2/\pi)(x - 0)$.

15. If $y = e^{rx}$, then $y' = re^{rx}$ and $y'' = r^2e^{rx}$. Thus

$$\begin{aligned} y'' - y' - 2y &= 0 \\ r^2e^{rx} - re^{rx} - 2e^{rx} &= 0 \\ e^{rx}(r^2 - r - 2) &= 0 \\ r^2 - r - 2 &= 0\ , \\ (r-2)(r+1) &= 0\ ,\end{aligned}$$

the solution of which is $r = -1$ and $r = 2$.

16. Let $x(t)$ be the position of car A at time t and let $y(t)$ be the position of car B at time t. Let $L(t)$ be the distance between car A and car B at time t. Then

$$\begin{aligned} L^2 &= x^2 + y^2 \\ 2L\frac{dL}{dt} &= 2x\frac{dx}{dt} + 2y\frac{dy}{dt} \\ \frac{dL}{dt} &= \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{L} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2+y^2}}\end{aligned}$$

Now $dx/dt = 60$ and $dy/dt = 80$, so when $x = 4$ and $y = 3$,

$$\frac{dL}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2+y^2}} = \frac{(4)(60) + (3)(80)}{\sqrt{(4)^2 + (3)^2}} = 96 \text{ mi/hr}$$

17. (a) If $f(x) = xe^{-(x^2/4)}$, then

$$\begin{aligned} f'(x) &= x\left(e^{-(x^2/4)}\right)' + e^{-(x^2/4)}(x)' \\ &= xe^{-(x^2/4)}\left(-\frac{x}{2}\right) + e^{-(x^2/4)} \\ &= e^{-(x^2/4)}\left(1 - \frac{x^2}{2}\right)\end{aligned}$$

(b) If $f(x) = x\cos(1/x^2)$, then

$$\begin{aligned} f'(x) &= x\left(\cos(1/x^2)\right)' + \left(\cos(1/x^2)\right)(x)' \\ &= x\left(-\sin(1/x^2)\right)\left(-2/x^3\right) + \cos(1/x^2) \\ &= \frac{2\sin(1/x^2)}{x^2} + \cos(1/x^2) \end{aligned}$$

18. If $\mathbf{r}(t) = \langle t^2\, ,\, 16t - 4t^2 \rangle$, then the velocity is $\mathbf{v}(t) = \langle 2t\, ,\, 16 - 8t \rangle$ and the acceleration is $\mathbf{a}(t) = \langle 2\, ,\, -8 \rangle$. Then $\mathbf{v}(t)$ and $\mathbf{a}(t)$ are orthogonal when

$$\begin{aligned} \mathbf{v}(t) \cdot \mathbf{a}(t) &= 0 \\ (2t)(2) + (16 - 8t)(-8) &= 0 \\ 4t - 128 + 64t &= 0 \\ 68t &= 128 \\ t &= \frac{32}{17} \end{aligned}$$