Fall 1998 Math 151 Common Exam 2

Multiple Choice Problems

1. $\lim_{x\to 0} \dfrac{2x}{\sin(3x)} = \dfrac{2}{3}$

2. $y = [2+\sec x]^{1/2}$ has derivative $y' = \frac{1}{2}[2+\sec x]^{-1/2}\sec x \tan x$, so the slope of the graph at $(\pi/3, 2)$ is $y'(\pi/3) = \sqrt{3}/2$.

3. $f(x) = e^{2x} + 2 + e^{-2x}$ and $f'(x) = 2(e^{2x} - e^{-2x})$.

4. $y = x\sin x + \cos x$ has second derivative $y'' = \cos x - x\sin x$

5. $A = \pi r^2 \Rightarrow \dfrac{dA}{dt} = 2\pi r\dfrac{dr}{dt} = 2\pi(10)2 = 40\pi$ ft/sec

6. $f(x) = 8(1+x)^{-1/2} \approx 8 - 4x + 3x^2$

7. $x_2 = x_1 - (x_1^3 - 2x_1 + 3)/(3x_1^2 - 2) = -1$ since $x_1 = 1$

8. $2 = (3y^2 - x)y' - y \Rightarrow 2 = 11y' - 2 \Rightarrow y' = 4/11$

9. $\lim_{x\to\infty} \dfrac{2e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}} = 2$

10. $g'(x) = 1/f'(2) = 1/4$

11. The solutions are $x = 1/2$ and $x = 4$; their product is 2.

Workout Problems

12. The slope is $y'(0) = e^0\cos 0 - e^0 \sin 0 = 1$. Using the point slope equation $y - y_0 = m(x - x_0)$, the tangent line is $y - 1 = 1(x-0)$ or $y = x+1$.

13. The given line has slope 1/2 and "tangent parallel" means $\dfrac{dy}{dx} = 1/2$. This gives the equation

$$\frac{dy}{dx} = \frac{dy}{dt}\Big/\frac{dx}{dt} = 2t/(3t^2+1) = 1/2.$$

Simplifying the last equality gives the quadratic

$$\begin{aligned} &4t = 3t^2 + 1, \quad \text{or} \\ &0 = 3t^2 - 4t + 1 = (3t-1)(t-1). \end{aligned}$$

The solutions $t = 1/3$ and $t = 1$ lead to the points (2,1) and (10/27, 1/9), respectively.

14. For any t velocity is $r'(t) = (2t-3)\bar{\imath} + t^{-2}\bar{\jmath}$ and acceleration is $r''(t) = 2\bar{\imath} - t^{-3}\bar{\jmath}$. Taking $t = 2$,

$$\begin{aligned} &\text{acceleration} = 2\bar{\imath} - (1/4)\bar{\jmath}, \text{ with} \\ &\text{magnitude} = [4 + (1/4)^2]^{1/2} = \sqrt{4.0625}\ \ m/s^2 \end{aligned}$$

15. For a sketch see Example 2 on page 216 of the text. Let x be the distance in feet from the bottom of the ladder to the wall, and y be the height in feet of the ladder's top.

$$\begin{aligned} 100 &= x^2 + y^2 \text{ by Pythogoras' Theorem} \\ 0 &= 2x\frac{dx}{dt} + 2y\frac{dy}{dt} \\ &= 2x\frac{2x}{dt} + 2\sqrt{100 - x^2}\ \frac{dy}{dt} \end{aligned}$$

Since $\frac{dx}{dt} = 2$ when $x = 8$,

$$0 = 2(8)(2) + 2(6)\frac{dy}{dt} \text{ or } \frac{dy}{dt} = -8/3 \text{ ft/sec.}$$

16. (a) $y = \ln(x+2) \Rightarrow e^y = x + 2 \Rightarrow f^{-1}(y) = x = e^y - 2$, or, after renaming the variable, $f^{-1}(x) = e^x - 2$.

(b) The domain of f^{-1} is the range of $f = (-\infty, \infty)$; the range of f^{-1} is the domain of $f = (-2, \infty)$.

17. $f'(x) = -2\sin x + 10[1 + \sin(x^2)]^9 \cos(x^2)(2x)$

18. (a) $h(5) = g(f(5)) = g(2) = 4$

(b) $h'(2) = g'(f(2))f'(2) = g'(1)(-1) = 2(-1)$