Section 1.1
Solutions and Hints

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for the book:
Calculus, Early Vectors
by James Stewart.

17. Find a unit vector which has the same direction as <1, 2>

Recall that a unit vector has length (or norm) of 1.
Also recall the vector <1, 2> has its tail at the origin: (0, 0).
The length of <1, 2> = $\sqrt{(1-0)^2 + (2-0)^2} = \sqrt{5}$

So if we divide all the components of <1, 2> by $\sqrt{5}$ we will have a unit vector which is parallel to <1, 2>.

Specifically we would get:

$$< \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} >$$

20. Find a unit vector which has the same direction as 2i – 4j

Note: 2**i** – 4**j** = <2, -4> and work this the same as problem 17.

The length of <2, -4> = $\sqrt{(2-0)^2 + (-4-0)^2} = \sqrt{20} = 2\sqrt{5}$

So if we divide all the components of <2, -4> by $2\sqrt{5}$ we will have a unit vector which is parallel to <2, -4>.

Specifically we would get:

$$< \frac{2}{2\sqrt{5}}, \frac{-4}{2\sqrt{5}} > = < \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}} >$$

21. Express i and j in terms of a and b.

Given: $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = \mathbf{i} - \mathbf{j}$.

Recall: $\mathbf{i} = \langle 1, 0\rangle$ and $\mathbf{j} = \langle 0, 1\rangle$

To express **i** in terms of **a** and **b** we must find constants m and n such that:

$$\begin{aligned}\langle 1, 0\rangle &= m*\mathbf{a} + n*\mathbf{b} \\ &= m*\langle 2, 3\rangle + n*\langle 1, -1\rangle \\ &= \langle 2m, 3m\rangle + \langle n, -n\rangle \\ &= \langle 2m + n, 3m - n\rangle\end{aligned}$$

So we have:

Eq. 1: $1 = 2m + n$ and

Eq. 2: $0 = 3m - n$

From Eq. 1 we get:

$$1 = 2m + n \quad \rightarrow 1 - 2m = n$$

And we substitute that into Eq. 2 to get:

$$\begin{aligned}0 = 3m - n \quad &\rightarrow 0 = 3m - (1 - 2m) \\ &\rightarrow 0 = 3m - 1 + 2m \\ &\rightarrow 1 = 5m \\ &\rightarrow m = 1/5\end{aligned}$$

Putting $m = 1/5$ into Eq. 1 we get:

$$\begin{aligned}1 = 2m + n \quad &\rightarrow 1 = 2*(1/5) + n \\ &\rightarrow 1 = 2/5 + n \\ &\rightarrow n = 3/5\end{aligned}$$

Thus we find that $\mathbf{i} = (1/5)\mathbf{a} + (3/5)\mathbf{b}$

To express **j** in terms of **a** and **b** we must find constants m and n such that:

$$\langle 0, 1\rangle = m*\mathbf{a} + n*\mathbf{b} = \langle 2m + n, 3m - n\rangle$$

So we have:

Eq. 3: $0 = 2m + n$ and

Eq. 4: $1 = 3m - n$

From Eq. 3 we get:

$$0 = 2m + n \quad \rightarrow -2m = n$$

And we substitute that into Eq. 4 to get:

$$\begin{aligned}1 = 3m - n \quad &\rightarrow 1 = 3m - (-2m) \\ &\rightarrow m = 1/5\end{aligned}$$

Putting $m = 1/5$ into Eq. 3 we get:

$$\begin{aligned}0 = 2m + n \quad &\rightarrow 0 = 2*(1/5) + n \\ &\rightarrow 0 = 2/5 + n \\ &\rightarrow n = -2/5\end{aligned}$$

Thus we find that $\mathbf{j} = (1/5)\mathbf{a} - (2/5)\mathbf{b}$

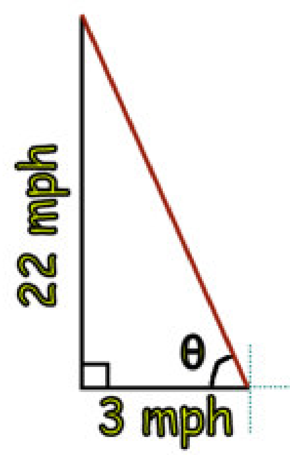
29. A woman walks due west on the deck of a ship at 3 miles per hour. The ship is moving north at a speed of 22 miles per hour. Find the speed and direction of the woman relative to the surface of the water.

Assume the water is not moving. If the woman stood still she would be moving 22 mph due north relative to the water. If the ship was not moving she would be moving due west at 3 mph relative to the water.

The first step to solve this problem is adding the vectors: $\langle -3, 0 \rangle$ and $\langle 0, 22 \rangle$

$$\langle -3, 0 \rangle + \langle 0, 22 \rangle = \langle -3, 22 \rangle$$

But speed is the magnitude of velocity and we need a direction in the form of North [so many degrees] West. To accomplish this you must use a triangle:



So the magnitude of $\langle -3, 22 \rangle = \text{sqrt}(9 + 484) \cong 22.2$ miles per hour.
And $\tan(\theta) = 22 / 3 \rightarrow \theta = \tan^{-1}(22/3) \rightarrow \theta \cong 82.2°$
Notice θ would be in terms of W 82.2° N, and we need N [blah°] W.
So we take $90 - 82.2 = 7.8°$ and we conclude:

She is moving about 22.2 mph at N 7.8° W.