Section 1.2
Solutions and Hints

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for the book:
Calculus, Early Vectors
by James Stewart.

31. Find a unit vector that is orthogonal to i + 3j.

Orthogonal and perpendicular mean the same thing.
Notice the 'slope' of the line from (0, 0) to (1, 3) is 3, thus something perpendicular to it would have a slope of –1/3.
Slope = rise/run = -1/3 means we might investigate the vector <-3, 1>

Check this by noting: <1, 3> • <-3, 1> = 1*(-3) + 3*1 = 0.

So now all we need is to make it a unit vector, which means it must have a magnitude of 1. So we will 'normalize' <-3, 1> by dividing it by its magnitude:

magnitude = | <-3, 1> | = sqrt(9 + 1) = sqrt(10) = $\sqrt{10}$

So our answer will be <-3, 1> / $\sqrt{10}$.

And we conclude:

> The unit vector $< -\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}} >$,
> is orthogonal to **i** + 3**j**.

You might also note that <3, -1> is also perpendicular to <1, 3>. Following the above steps you will arrive at the book's answer if you start with this vector. Notice they are virtually the same answer, except one points up left and the other points down right.

35. Find the scalar and vector projections of b onto a.

Given: **a** = <4, 2> and **b** = <1, 1>

<u>The scalar projection (or component) of **b** onto **a**</u>:

$\text{comp}_{\mathbf{a}}\mathbf{b} = (\mathbf{a} \bullet \mathbf{b}) / |\mathbf{a}|$

$= (4*1 + 2*1) / \text{sqrt}(16 + 4)$

$= \dfrac{6}{2\sqrt{5}}$ $\boxed{= \dfrac{3}{\sqrt{5}}}$

<u>The vector projection (or just projection) of **b** onto **a**</u>:

$\text{proj}_{\mathbf{a}}\mathbf{b} = \left(\dfrac{a \bullet b}{|a|}\right) * \dfrac{a}{|a|}$, note the part in parentheses is the scalar projection

so we can just put in $\dfrac{3}{\sqrt{5}}$ and get

$= \dfrac{3}{\sqrt{5}} * \dfrac{<4,2>}{2\sqrt{5}}$

$= \dfrac{<12,6>}{10}$

$= <\dfrac{12}{10}, \dfrac{6}{10}>$ $\boxed{= <\dfrac{6}{5}, \dfrac{3}{5}>}$

41. Find $\text{proj}_{\mathbf{a}\perp}\mathbf{b}$.

Given **a** = <-2, 3>, **b** = <0, 2>

By definition: $\mathbf{a}^{\perp}$ = <-3, -2>

So $\text{proj}_{\mathbf{a}\perp}\mathbf{b} = \dfrac{a^{\perp} \bullet b}{|a^{\perp}|^2} a^{\perp} = \dfrac{-3*0 + -2*2}{\left(\sqrt{(-3)^2 + (-2)^2}\right)^2} * <-3,-2>$

$= \dfrac{-4}{13} <-3,-2>$

$\boxed{\text{proj}_{\mathbf{a}\perp}\mathbf{b} = <\dfrac{12}{13}, \dfrac{8}{13}>}$

42. Find $\text{proj}_{\mathbf{a}\perp}\mathbf{b}$.

Given $\mathbf{a}$ = <-3, 2>, $\mathbf{b}$ = <1, 2>

By definition: $\mathbf{a}^{\perp}$ = <-2, -3>

So $\text{proj}_{\mathbf{a}\perp}\mathbf{b}$ $= \dfrac{a^{\perp} \bullet b}{|a^{\perp}|^2} a^{\perp} = \dfrac{-2*1+-3*2}{\left(\sqrt{(-2)^2+(-3)^2}\right)^2} * <-2,-3>$

$= \dfrac{-8}{13} <-2,-3>$

$\text{proj}_{\mathbf{a}\perp}\mathbf{b} = <\dfrac{16}{13}, \dfrac{24}{13}>$

43. Find the distance from the point (3, 7) to the line y = 2x.

Pick two points that are on the line – to do this you usually pick either 0 or 1 and substitute it in for x and solve for y, or sub it in for y and solve for x.

Let x = 0 → y = 2*0 → y = 0 So **(0, 0)** is on the line.
Let x = 1 → y = 2*1 → y = 2 So **(1, 2)** is on the line.

Now we find a vector that is parallel to (and on) the line:
Let $\mathbf{a}$ = <1, 2> – <0, 0> = <1, 2>

From this we know that $\mathbf{a}^{\perp}$ = < -2, 1 > is perpendicular to the line.

We also need a vector that has its head at (3,7) and its tail somewhere on the line.
Let $\mathbf{b}$ = <3, 7> – <0, 0> = <3, 7>

Now we know that $|\text{comp}_{\mathbf{a}\perp}\mathbf{b}|$ = the distance from the point (3, 7) to the line.

$|\text{comp}_{\mathbf{a}\perp}\mathbf{b}| = \left|\dfrac{a^{\perp} \bullet b}{|a^{\perp}|}\right| = \left|\dfrac{<-2,1> \bullet <3,7>}{\sqrt{(-2)^2+1^2}}\right|$

$= \left|\dfrac{-2*3+1*7}{\sqrt{5}}\right|$

$= \left|\dfrac{1}{\sqrt{5}}\right|$

So the distance from the point (3, 7) to the line y = 2x is $\dfrac{1}{\sqrt{5}}$

51. If a = <3, -1>, find a vector b such that comp$_a$b = 2.

By definition comp$_{\mathbf{a}}$**b** = $\frac{a \bullet b}{|a|}$. Denote **b** = <x, y>.

Thus

$$2 = \frac{<3,-1> \bullet b}{\sqrt{3^2 + (-1)^2}} = \frac{<3,-1> \bullet <x, y>}{\sqrt{10}} = \frac{3x - y}{\sqrt{10}}$$

So we have:

$2\sqrt{10} = 3x - y$. Or rather $y = 3x - 2\sqrt{10}$.

Let s be some constant. The $y = 3s - 2\sqrt{10}$.
And we conclude:

> any vector **b** = <s, $3s - 2\sqrt{10}$ > will satisfy the requirement

A specific answer would be to let s = 0.
Then **b** = <0, $-2\sqrt{10}$ > would satisfy comp$_{\mathbf{a}}$**b** = 2 with **a** = <3, -1>.