Section 1.3
Solutions and Hints

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for the book:
<u>Calculus, Early Vectors</u>
by James Stewart.

25. Find a vector equation and a parametric equation for the line passing through (1, 3) and (-2, 7).

<u>Finding the vector equation</u>:

By definition the vector equation of a line is: $\mathbf{r}(t) = \mathbf{r_0} + t*\mathbf{v}$,
where t is a scalar variable (usually time) and $\mathbf{r_0}$ and $\mathbf{v}$ are a vectors with their heads on the given line. Additionally $\mathbf{v}$ must have its tail also on the line (or be parallel with the line. What this means is that $\mathbf{r_0}$ and $\mathbf{v}$ are basically formed from 2 points on the line.

To find a vector equation of a line you must have 2 points, P_0 and P on the line.
We are given two points on the line.
$\mathbf{r_0}$ will be from (0, 0) to a point P_0 on the line.
a will be from P_0 to another point on the line.
a will be rewritten as the vector $t*\mathbf{v}$, where t is a variable scalar.
(Think of **a** as being a specific case of $t*\mathbf{v}$, where $t = 1$)

We let $\mathbf{r_0}$ = <1, 3> - <0, 0> = <1, 3> and **a** = <-2, 7> - <1, 3> = <-3, 4>

And we arrive at:

$\mathbf{r}(t)$ = <1, 3> + t*<-3, 4>

<u>Finding the parametric equation</u>:

From the vector form we get:

$\mathbf{r}(t)$ = < x(t), y(t) > = <1, 3> + t*<-3, 4> = <1 – 3t, 3 + 4t>

Or rather:

$x(t) = 1 - 3t$ and $y(t) = 3 + 4t$.

26. Find a vector equation and a parametric equation for the line passing through (-3, 4) and (2, 8).

Finding the vector equation:

By definition the vector equation of a line is: $\mathbf{r}(t) = \mathbf{r_0} + t*\mathbf{v}$,
where t is a scalar variable (usually time) and $\mathbf{r_0}$ and $\mathbf{v}$ are a vectors with their heads on the given line. Additionally $\mathbf{v}$ must have its tail also on the line (or be parallel with the line. What this means is that $\mathbf{r_0}$ and $\mathbf{v}$ are basically formed from 2 points on the line.

To find a vector equation of a line you must have 2 points, P_0 and P on the line.
We are given two points on the line.
$\mathbf{r_0}$ will be from (0, 0) to a point P_0 on the line.
$\mathbf{a}$ will be from P_0 to another point on the line.
$\mathbf{a}$ will be rewritten as the vector $t*\mathbf{v}$, where t is a variable scalar.
(Think of $\mathbf{a}$ as being a specific case of $t*\mathbf{v}$, where $t = 1$)

We let $\mathbf{r_0} = <-3, 4> - <0, 0> = <-3, 4>$ and $\mathbf{a} = <2, 8> - <-3, 4> = <5, 4>$

And we arrive at:

$$\mathbf{r}(t) = <-3, 4> + t*<5, 4>$$

Finding the parametric equation:

From the vector form we get:

$$\mathbf{r}(t) = < x(t), y(t) > = <-3, 4> + t*<5, 4> = <-3 + 5t, \ 4 + 4t>$$

Or rather:

$$x(t) = -3 + 5t \quad \text{and} \quad y(t) = 4 + 4t.$$

29. Find a vector equation, a parametric equation and a Cartesian equation for the line passing through P(-4, 5) and parallel to a = <-2, 6>

<u>Finding the vector equation</u>:

By definition the vector equation of a line is: $\mathbf{r}(t) = \mathbf{r_0} + t*\mathbf{v}$,
where t is a scalar variable (usually time) and $\mathbf{r_0}$ and $\mathbf{v}$ are a vectors with their heads on the given line. Additionally $\mathbf{v}$ must have its tail also on the line (or be parallel with the line. What this means is that $\mathbf{r_0}$ and $\mathbf{v}$ are basically formed from 2 points on the line.

To find a vector equation of a line you must have 2 points, P_0 and P on the line.
We are given one point on the line and a vector parallel to the line.
$\mathbf{r_0}$ will be from (0, 0) to a point P_0 on the line.
a will be the parallel vector.
a will be rewritten as the vector $t*\mathbf{v}$, where t is a variable scalar.
(Think of **a** as being a specific case of $t*\mathbf{v}$, where $t = 1$)

We let $\mathbf{r_0} = <-4, 5> - <0, 0> = <-4, 5>$ and $\mathbf{a} = <-2, 6>$

And we arrive at:

$$\mathbf{r}(t) = <-4, 5> + t*<-2, 6>$$

<u>Finding the parametric equation</u>:

From the vector form we get:

$$\mathbf{r}(t) = < x(t), y(t) > = <-4, 5> + t*<-2, 6> = <-4 - 2t, 5 + 6t>$$

Or rather:

$$x(t) = -4 - 2t \quad \text{and} \quad y(t) = 5 + 6t.$$

<u>Finding the Cartesian equation</u>:

From the parametric equation we get:

$$x = -4 - 2t \rightarrow x + 4 = -2t \rightarrow -(x + 4)/2 = t \rightarrow t = (-x - 4)/2$$

Putting that in for t in the y(t) equation we arrive at:

$$y = 5 + 6*[(-x - 4) / 2]$$
$$\rightarrow y = 5 - 3x - 12$$
$$\rightarrow y = -3x - 7$$

So the Cartesian equation is:

$$y = -3x - 7$$

30. Find a vector equation, a parametric equation and a Cartesian equation for the line passing through P(2, 5) and parallel to a = <3, 0>

Finding the vector equation:

By definition the vector equation of a line is: $\mathbf{r}(t) = \mathbf{r_0} + t^*\mathbf{v}$,
where t is a scalar variable (usually time) and $\mathbf{r_0}$ and $\mathbf{v}$ are a vectors with their heads on the given line. Additionally $\mathbf{v}$ must have its tail also on the line (or be parallel with the line. What this means is that $\mathbf{r_0}$ and $\mathbf{v}$ are basically formed from 2 points on the line.

To find a vector equation of a line you must have 2 points, P_0 and P on the line.
We are given one point on the line and a vector parallel to the line.
$\mathbf{r_0}$ will be from (0, 0) to a point P_0 on the line.
a will be the parallel vector.
a will be rewritten as the vector $t^*\mathbf{v}$, where t is a variable scalar.
(Think of **a** as being a specific case of $t^*\mathbf{v}$, where $t = 1$)

We let $\mathbf{r_0} = \langle 2, 5\rangle - \langle 0, 0\rangle = \langle 2, 5\rangle$ and $\mathbf{a} = \langle 3, 0\rangle$

And we arrive at: $\mathbf{r}(t) = \langle 2, 5\rangle + t^*\langle 3, 0\rangle$

Finding the parametric equation:

From the vector form we get:

$$\mathbf{r}(t) = \langle x(t), y(t) \rangle = \langle 2, 5\rangle + t^*\langle 3, 0\rangle = \langle 2 + 3t, 5\rangle$$

Or rather:

$$x(t) = 2 + 3t \quad \text{and} \quad y(t) = 5.$$

Finding the Cartesian equation:

Notice in this case $y(t)$ is NOT dependent on the value of x (or t), in fact $y(t)$ is constantly 5.

So the Cartesian equation is: $y = 5$