Section 2.3
Solutions and Hints

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for the book:
Calculus, Early Vectors
by James Stewart.

42. Calculate the given limit:

$\lim_{t \to 1} \mathbf{r}(t)$, where $\mathbf{r}(t) = \left\langle \frac{t^3 - 1}{t^2 - 1}, \frac{t^2 - 2t + 1}{t - 1} \right\rangle$ = < x(t), y(t) >

For vectors you take the limit of each component separately.
In this case we will first do the x(t) part, then the y(t) part.

First notice that: $(t^3 - 1) / (t^2 - 1)$ can be simplified by factoring out stuff:

$t^3 - 1 = (t - 1)(t^2 + t + 1)$
$t^2 - 1 = (t - 1)(t + 1)$

Thus $\frac{t^3 - 1}{t^2 - 1} = \frac{(t-1)(t^2 + t + 1)}{(t-1)(t+1)} = \frac{t^2 + t + 1}{t + 1}$

So $\lim_{t \to 1} \mathbf{x}(t) = \lim_{t \to 1} \frac{t^2 + t + 1}{t + 1} = (1 + 1 + 1) / (1 + 1) =$ **<u>3/2</u>**

Likewise factoring will assist us in with the $\lim_{t \to 1} \mathbf{y}(t)$.
$t^2 - 2t + 1 = (t - 1)(t - 1)$

Thus $\frac{t^2 - 2t + 1}{t - 1} = \frac{(t-1)(t-1)}{(t-1)} = (t - 1)$

So $\lim_{t \to 1} \mathbf{y}(t) = \lim_{t \to 1} (t - 1) =$ **<u>0</u>**

And we conclude:

$\lim_{t \to 1} \mathbf{r}(t) = <3/2, 0$

55. Calculate $\lim_{x \to 2}\left(\frac{|x-2|}{x-2}\right)$

For this one (and almost any limit involving absolute values) we need to break things into "from the left" and "from the right." If the limit exists everything will converge to the same value.

$$\lim_{x \to 2+}\left(\frac{|x-2|}{x-2}\right) = \lim_{x \to 2+}\left(\frac{x-2}{x-2}\right)$$ (because: if x > 2 then x – 2 will always be positive)

$$= \lim_{x \to 2+}(1) \quad = \quad \underline{\mathbf{1}}$$

$$\lim_{x \to 2-}\left(\frac{|x-2|}{x-2}\right) = \lim_{x \to 2-}\left(\frac{-(x-2)}{x-2}\right)$$ (because: if x < 2 then x – 2 will always be negative)

$$= \lim_{x \to 2+}(-1) \quad = \quad \underline{\mathbf{-1}}$$

Notice the limit from the left does not equal the limit from the right, $-1 \neq 1$.
So we conclude:

The limit Does Not Exist (DNE).

71. Let F(x) = $\left(\frac{x^2-1}{|x-1|}\right)$

71 a i. Find the limit of F(x) as x → 1 from the right.

$$\lim_{x\to 1+}\left(\frac{x^2-1}{|x-1|}\right) = \lim_{x\to 1+}\left(\frac{x^2-1}{x-1}\right),$$ because if x > 1, then x – 1 is positive

$$= \lim_{x\to 1+}\left(\frac{(x-1)(x+1)}{x-1}\right)$$

$$= \lim_{x\to 1+}(x+1)$$

$$= \boxed{2}$$

71 a ii. Find the limit of F(x) as x → 1 from the left.

$$\lim_{x\to 1-}\left(\frac{x^2-1}{|x-1|}\right) = \lim_{x\to 1-}\left(\frac{x^2-1}{-(x-1)}\right),$$ because if x < 1, then x – 1 is positive

$$= \lim_{x\to 1-}\left(\frac{(x-1)(x+1)}{-(x-1)}\right)$$

$$= \lim_{x\to 1-}-(x+1)$$

$$= \boxed{-2}$$

71 b. Does lim$_{x\text{-->}1}$ F(x) exist?

No, the limit does NOT exist,
because the limit from the right = 2
and the limit from the left = -2, and 2 ≠ -2.

Part c is left for you to complete.