# Section 2.5
## Solutions and Hints

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**for the book:**
**<u>Calculus, Early Vectors</u>**
**by James Stewart.**

**29. Let f(x) be defined as below. Show that f is continuous on (-∞, ∞).**

$$f(x) = \begin{cases} x - 1 & x < 3 \\ 5 - x & x \geq 3 \end{cases}$$

Clearly x – 1 is continuous from (-∞, 3), as is 5 – x from [3, ∞)
So the only point we need to worry about is at x = 3.

To show f(x) is continuous at x = 3, we need to show that:
$\lim_{x \to 3} f(x) = f(3)$

To find $\lim_{x \to 3} f(x)$ we must find the left AND right limits.

<u>The left limit</u>:
$\lim_{x \to 3^-} (x - 1) = 3 - 1 = 2$ ( you ought to use tables )

<u>The right limit</u>:
$\lim_{x \to 3^+} (5 - x) = 5 - 3 = 2$ ( you ought to use tables )

Thus we know: $\lim_{x \to 3} f(x) = 2$

And f(3) = 5 – 3 = 2

So we have: $\lim_{x \to 3} f(x) = 2 = f(3)$, thus f(x) is continuous.

**38. Find the constant *c* that makes g(x) continuous on (-∞, ∞).**

$$g(x) = \begin{cases} x^2 - c^2 & x < 4 \\ cx + 20 & x \geq 4 \end{cases}$$

For any $c$, $x^2 - c^2$ will be continuous on (-∞, 4).
For any $c$, $c$x + 20 will be continuous on [4, ∞).
So the only point we need to worry about is at x = 4.
For this we need to find a $c$ such that the $\lim_{x \to 4-} = \lim_{x \to 4+} = f(4)$.
To accomplish this all we need to do is find $c$ such that:
$4^2 - c^2 = c*4 + 20$ → $0 = c^2 + 4c + 4$
→ $0 = (c + 2)(c + 2)$

Thus $c = -2$ will make g(x) continuous on (-∞, ∞).

**39. Find the constants *c* and *d* that makes h(x) continuous on (-∞, ∞).**

$$g(x) = \begin{cases} 2x & x < 1 \\ cx^2 + d & 1 \leq x \leq 2 \\ 4x & x > 2 \end{cases}$$

For any $c$ and $d$, 2x will be continuous on (-∞, 1).
For any $c$ and $d$, $cx^2 + d$ will be continuous on [1, 2].
For any $c$ and $d$, 4x will be continuous on (2, ∞)
So the points we need to worry about are at x = 1 and x = 2
For this we need to find a $c$ and $d$ such that
$\lim_{x \to 1-} = \lim_{x \to 1+} = f(1)$ AND $\lim_{x \to 2-} = \lim_{x \to 2+} = f(2)$

To accomplish this we need to find $c$ and $d$ such that:
Equation 1: $2*1 = c*1^2 + d$ → $\mathbf{2 = c + d}$ AND
Equation 2: $c*2^2 + d = 4*2$ → $\mathbf{4c + d = 8}$ are both true.

Solving for $d$ using equation 1 we get:
$2 = c + d$ → $2 - c = d$

Substitute this in for $d$ into equation 2 and we get:
$4c + d = 8$ → $4c + (2 - c) = 8$, and we solve for $c$.
→ $3c = 6$
→ $\underline{c = 2}$

Putting $c = 2$ back into equation 1 we get:
$2 = c + d$ → $2 = 2 + d$ → $\underline{0 = d}$

Thus we conclude:

$c = 2$ and $d = 0$

**43. If $f(x) = x^3 - x^2 + x$, show there is a number *c* such that $f(c) = 10$.**

f(x) is clearly continuous on $(-\infty, \infty)$, specifically on [0, 3]
$f(0) = 0$
$f(3) = 27 - 9 + 3 = 21$

$f(0) = 0 < 10 < 21 = f(3)$, by the Intermediate Value Theorem there must exist a number $c \in (0, 3)$ such that $f(c) = 10$.

**47. Use the Intermediate Value Theorem (IVT) to show that there is a root of $x^3 + 2x = x^2 + 1$ in the interval (0, 1).**

Let $f(x) = x^3 - x^2 + 2x - 1$.
For there to exist a root of f(x) in (0, 1) there must exist some number $c \in (0, 1)$ such that $f(c) = 0$.

Clearly f(x) is continuous on [0, 1].
$f(0) = -1$
$f(1) = 1 - 1 + 2 - 1 = 1$

$f(0) = -1 < 0 < 1 = f(1)$ thus by the IVT there exists a number $c \in (0, 1)$ such that $f(c) = 0$. Thus there must be a root to the given equation in the interval (0, 1).