Section 2.6
Solutions and Hints

by Brent M. Dingle

for the book:
<u>Calculus, Early Vectors</u>
by James Stewart.

34. Find the horizontal and vertical asymptotes for the given curve.

$$y = \frac{x^2+4}{x^2-1} \qquad = \frac{x^2+4}{(x-1)(x+1)}$$

Be setting the denominator equal to zero we will find the vertical asymptotes:

x – 1 = 0 or x + 1 = 0
x = 1 or x = -1

So there are <u>vertical asymptotes at x = -1 and x = 1</u>.

To find the horizontal asymptotes we must take the limit as x → ∞ and the limit as x → -∞.

$$\lim_{x\to\infty}\frac{x^2+4}{x^2-1} \qquad = \lim_{x\to\infty}\frac{\left(\frac{x^2}{x^2}+\frac{4}{x^2}\right)}{\left(\frac{x^2}{x^2}-\frac{1}{x^2}\right)},$$ divide everything by x^2.

$$= \lim_{x\to\infty}\frac{\left(1+\frac{4}{x^2}\right)}{\left(1-\frac{1}{x^2}\right)},$$ apply limit law 5, from page 91, section 2.3

Problem 34, continued from previous page:

$$= \frac{\lim_{x\to\infty} 1 + \frac{4}{x^2}}{\lim_{x\to\infty} 1 - \frac{1}{x^2}},$$ apply limit laws 1 and 2, from section 2.3

$$= \frac{\lim_{x\to\infty} 1 + \lim_{x\to\infty} \frac{4}{x^2}}{\lim_{x\to\infty} 1 - \lim_{x\to\infty} \frac{1}{x^2}} = \frac{1+0}{1-0} = \underline{1}$$

Notice since we are dealing strictly with x^2 whether we go towards +∞ of -∞ does not change things as far as the limit is concerned. Specifically we should see:

$$\lim_{x\to-\infty} \frac{x^2+4}{x^2-1} = \frac{\lim_{x\to-\infty} 1 + \lim_{x\to-\infty} \frac{4}{x^2}}{\lim_{x\to-\infty} 1 - \lim_{x\to-\infty} \frac{1}{x^2}} = \frac{1+0}{1-0} = \underline{1}$$

So the limits are the same and there is only ONE horizontal asymptote at y = 1.

Thus our final answer is:

horizontal asymptotes at: x = 1 and x = -1 vertical asymptote ay: y = 1

35. Find the horizontal and vertical asymptotes for the given curve.

$$y = \frac{x^3}{x^2 + 3x - 10} = \frac{x^3}{(x+5)(x-2)}$$

So by setting the denominator equal to zero we find that there are vertical asymptotes at x = -5, and x =2.

To find the horizontal asymptotes we must find the limits as x→∞ and x→-∞.

$$\lim_{x\to\infty} \frac{x^3}{x^2 + 3x - 10} \qquad = \lim_{x\to\infty} \frac{\frac{x^3}{x^2}}{\frac{x^2}{x^2} + \frac{3x}{x^2} - \frac{10}{x}}, \quad \text{divide everything by } x^2$$

$$= \frac{\lim\limits_{x\to\infty} x}{\lim\limits_{x\to\infty} 1 + 3/x - 10/x}$$

$$= \frac{\infty}{1 + 0 + 0} = \infty$$

Notice if we simply change the x→∞ to x→-∞ we would arrive at -∞. So the limit as x→ ∞ = ∞ and the limit x→-∞ = -∞. Thus there are NO horizontal asymptotes.

Our final answer is then:

There are vertical asymptotes at x = -5 and x = 2.
There are NO horizontal asymptotes.