

Section 2.7

Solutions and Hints

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for the book:
Calculus, Early Vectors
by James Stewart.

11. If a ball is thrown in the air with a velocity of 40 feet per second, its height, in feet, after t seconds is given by $y = 40t - 16t^2$. Find the velocity at $t = 2$.

So let $f(x) = y = 40t - 16t^2$.

To determine the velocity at time, $t = 2$, you should apply the formula:

$$\begin{aligned}v(2) &= \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \rightarrow 0} \frac{(40*(2+h) - 16*(2+h)^2) - (40*2 - 16*2^2)}{h} \\&= \lim_{h \rightarrow 0} \frac{(80 + 40h - 16*(4 + 4h + h^2)) - (80 - 64)}{h} \\&= \lim_{h \rightarrow 0} \frac{(80 + 40h - 64 - 64h - 16h^2) - 16}{h} \\&= \lim_{h \rightarrow 0} \frac{(16 - 24h - 16h^2) - 16}{h} \\&= \lim_{h \rightarrow 0} \frac{-24h - 16h^2}{h} \\&= \lim_{h \rightarrow 0} -24 - 16h \\&= \lim_{h \rightarrow 0} (-24) - \lim_{h \rightarrow 0} 16h \\&= -24 - 0 \\&= -24 \text{ feet per second.}\end{aligned}$$

At time = 2 seconds,
the ball is moving at -24 ft/sec.

23. If a ball is thrown in the air with a velocity of $10\mathbf{i} + 30\mathbf{j}$ feet per second, its position after t seconds is given by:

$$\mathbf{r}(t) = 10t\mathbf{i} + (30t - 16t^2)\mathbf{j}$$

23a. Find the velocity of the ball when $t = 1$.

For this you apply the formula:

$$\begin{aligned} \mathbf{v}(1) &= \lim_{h \rightarrow 0} \frac{\mathbf{r}(1+h) - \mathbf{r}(1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(10 * (1+h)\mathbf{i} + (30 * (1+h) - 16 * (1+h)^2)\mathbf{j}) - (10 * 1\mathbf{i} + (30 * 1 - 16 * 1^2)\mathbf{j})}{h} \\ &= \lim_{h \rightarrow 0} \frac{((10 + 10h)\mathbf{i} + (30 + 30h - 16 - 32h - 16h^2)\mathbf{j}) - (10\mathbf{i} + 14\mathbf{j})}{h} \\ &= \lim_{h \rightarrow 0} \frac{(10\mathbf{i} + 10h\mathbf{i} + 30\mathbf{j} + 30h\mathbf{j} - 16\mathbf{j} - 32h\mathbf{j} - 16h^2\mathbf{j}) - (10\mathbf{i} + 14\mathbf{j})}{h} \\ &= \lim_{h \rightarrow 0} \frac{10\mathbf{i} + 10h\mathbf{i} + 14\mathbf{j} + 30h\mathbf{j} - 32h\mathbf{j} - 16h^2\mathbf{j} - 10\mathbf{i} - 14\mathbf{j}}{h} \\ &= \lim_{h \rightarrow 0} \frac{10h\mathbf{i} - 2h\mathbf{j} - 16h^2\mathbf{j}}{h} \\ &= \lim_{h \rightarrow 0} (10\mathbf{i} - 2\mathbf{j} - 16h\mathbf{j}) \\ &= 10\mathbf{i} - 2\mathbf{j} - 0 \end{aligned}$$

Velocity at time, $t = 1$, is $10\mathbf{i} - 2\mathbf{j}$.

23b. What is the speed of the ball when $t = 1$.

This is simply the magnitude of the vector $10\mathbf{i} - 2\mathbf{j} = \langle 10, -2 \rangle$

$$|\langle 10, -2 \rangle| = \sqrt{10^2 + (-2)^2} = \sqrt{104}$$

So the ball is traveling at $\sqrt{104}$ ft/sec

To compare this problem with problem 11, you may want to solve for velocity and speed when $t = 2$ and see if you get the same result (you won't by the way), but maybe you can figure out why not (compare initial velocities).