Section 3.1
Solutions and Hints

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for the book:
Calculus, Early Vectors
by James Stewart.

23. Find the derivative of the given function using the definition of derivative. State the domain of both the function and its derivative.

$$g(x) = \sqrt{1+2x}$$

Notice the domain of g(x) is found by solving (1 + 2x) ≥ 0,
which gives 2x ≥ -1 → x ≥ -½, or rather x ∈ [-½, ∞).

By the definition of derivative: $g'(x) = \lim_{h\to 0} \dfrac{g(x+h)-g(x)}{h}$

$$g'(x) = \lim_{h\to 0} \frac{\sqrt{1+2*(x+h)} - \sqrt{1+2x}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{1+2x+2h} - \sqrt{1+2x}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{1+2x+2h} - \sqrt{1+2x}}{h} * \frac{\sqrt{1+2x+2h} + \sqrt{1+2x}}{\sqrt{1+2x+2h} + \sqrt{1+2x}}$$

$$= \lim_{h\to 0} \frac{(1+2x+2h)-(1+2x)}{h*\sqrt{1+2x+2h} + \sqrt{1+2x}}$$

$$= \lim_{h\to 0} \frac{2h}{h*\sqrt{1+2x+2h} + \sqrt{1+2x}}$$

$$= \lim_{h\to 0} \frac{2}{\sqrt{1+2x+2h} + \sqrt{1+2x}},$$ now "put in" zero for h

$$= \frac{2}{\sqrt{1+2x} + \sqrt{1+2x}} = \frac{2}{2*\sqrt{1+2x}} = \frac{1}{\sqrt{1+2x}}$$

Which has domain found by (1 + 2x) > 0 → x > -½, or x ∈ (-½, ∞)

Thus $g'(x) = \dfrac{1}{\sqrt{1+2x}}$.
And the domain of g(x) is [-½,∞) and the domain of g'(x) is (-½,∞).

26. Find the derivative of the given function using the definition of derivative. State the domain of both the function and its derivative.

$$g(x) = \frac{1}{x^2}$$

Notice the domain of g(x) is found by solving $x^2 \neq 0$,
which gives $x \neq 0$, or rather $x \in (-\infty, 0) \cup (0, \infty)$.

By the definition of derivative: $g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$

$$g'(x) = \lim_{h \to 0} \frac{\left(\frac{1}{(x+h)^2}\right) - \left(\frac{1}{x^2}\right)}{h}$$, Think about $\frac{\left(\frac{1}{49}\right) - \left(\frac{1}{16}\right)}{3}$

How would you solve it (without a calculator)? First you would find a common denominator for 1/49 and 1/16. Then you would subtract. Then you would divide by 3. We will do similar.

$$g'(x) = \lim_{h \to 0} \frac{\left(\frac{x^2}{x^2(x+h)^2}\right) - \left(\frac{(x+h)^2}{x^2(x+h)^2}\right)}{h}$$, common denominator

$$= \lim_{h \to 0} \frac{\left(\frac{x^2}{x^2(x+h)^2}\right) - \left(\frac{x^2 + 2xh + h^2}{x^2(x+h)^2}\right)}{h}$$, expand stuff out

$$= \lim_{h \to 0} \frac{\left(\frac{x^2 - x^2 - 2xh - h^2}{x^2(x+h)^2}\right)}{h}$$, simplify a little

$$= \lim_{h \to 0} \left(\frac{-2xh - h^2}{x^2(x+h)^2}\right) * \frac{1}{h}$$, we changed ‘divided by h’ to ‘times 1/h’

$$= \lim_{h \to 0} \frac{(-2x - h) * h}{x^2(x+h)^2} * \frac{1}{h}$$, pull an h out of the numerator and cancel it

$$= \lim_{h \to 0} \frac{(-2x - h)}{x^2(x+h)^2}$$, “put in” zero for h

$$= \frac{-2x - 0}{x^2(x+0)^2} = \frac{-2x}{x^4} = \frac{-2}{x^3}$$

Which has domain found by solving $x^3 \neq 0$.
Which gives $x \neq 0$, or rather $x \in (-\infty, 0) \cup (0, \infty)$.

Thus $g'(x) = \frac{-2}{x^3}$
and g(x) and g’(x) both have domain $x \in (-\infty, 0) \cup (0, \infty)$.