Section 3.3
Solutions and Hints

by Brent M. Dingle

for the book:
Calculus, Early Vectors
by James Stewart.

The entire point of this section is that **Instantaneous Rates of Change are derivatives**. Every now and then one of the professors gets aggressive and thinks all students should know **Marginal Cost is the derivative of the Cost function**. In fact marginal anything is the derivative of the anything function.

You also need to understand that the derivative MAY be approximated by the Average Rate of Change. This is often done when the actual function is NOT known and all you have is a table of data (like temperature every 10 seconds). On top of that notice that your estimate (usually) improves as your step size decreases.

11. A stone is dropped into a lake which creates a circular ripple that travels outward at a speed of 60 cm per second. Find the rate at which the area within the circle is increasing after 1 second, 3 seconds and 5 seconds.

The only trick to this one is to know the area of a circle = A = $\pi * r^2$ and that $r(t) = 60*t$. Thus $A(t) = \pi * (60*t)^2 = 3600\pi * t^2$.

Given that $A(t) = 3600\pi * t^2$ we know the rate of change of the area will be $A'(t)$.

$A'(t) = 3600\pi * 2* t^{2-1} = 7200\pi * t$.

And to answer the question all we need to do is find A'(1), A'(3) and A'(5)

$A'(1) = 7200\pi*1 =$ 7200π cm^2/s

$A'(3) = 7200\pi*3 =$ $21{,}600\pi$ cm^2/s

$A'(5) = 7200\pi*5 =$ $36{,}00\pi$ cm^2/s

24. The population of a slowly growing bacterial colony after *t* hours is given by $n = 100 + 24t + 2t^2$. Find the growth rate after 2 hours.

Note that everything is measuring time in hours so there is no need to worry about changing stuff to seconds – this will not always be the case.

Given $n(t) = 100 + 24t + 2t^2$, the growth rate will be $n'(t)$

$n'(t) = 24 + 4t$

So the growth rate after 2 hours is $n'(2) = 24 + 4*2 = 32$.

The growth rate after 2 hours is 32.