Section 3.4 (5.7)
Solutions and Hints

by Brent M. Dingle

for the book:
Calculus, Early Vectors
by James Stewart.

7. Find the limit of the given function.

Obviously putting zero in for t results in dividing by zero, which is not good. So we must find something else to do. We "know" that $\lim_{x \to 0} [(\sin x)/ x] = 1$. Can we get our function into that form? Specifically can we get our function into the form: that $\lim_{t \to 0} [(\sin 5t)/ 5t]$. The answer is yes. Recall that 1 / (1/5) = 5. So if we "take out" 1/5 from the denominator then good things should happen. If you prefer think of it as multiplying both the numerator and denominator by 5.

$$\lim_{t \to 0} \frac{\sin 5t}{t} = \lim_{t \to 0} \frac{\sin 5t}{t} * \frac{5}{5}$$

$$= \lim_{t \to 0} \frac{5 * \sin 5t}{5t},$$ recall limit law 3 from section 2.3 which allows us to take constants out of limits

$$= 5 * \lim_{t \to 0} \frac{\sin 5t}{5t},$$ Thm 4 of this section: $\lim_{x \to 0} [(\sin x)/ x] = 1$

$= 5 * 1$

$= 5$

So from the above: $\lim_{t \to 0} \frac{\sin 5t}{t} = 5$

10. Find the limit of the given function.

First notice if we just "put zero in" for x, we end up dividing by zero. So we must try something else. To that end we remember $\cos^2(x) + \sin^2(x) = 1$.
We then think… hmmm, what happens if we "multiply by one" or rather we multiply both the numerator and the denominator by cos(x) + 1…

$$\lim_{x\to 0}\frac{\cos(x)-1}{\sin(x)} = \lim_{x\to 0}\frac{\cos(x)-1}{\sin(x)} * \frac{\cos(x)+1}{\cos(x)+1}$$

$$= \lim_{x\to 0}\frac{\cos^2(x)-1}{\sin(x)*\left(\cos(x)+1\right)},$$ remember $\cos^2(x) + \sin^2(x) = 1$.

$$= \lim_{x\to 0}\frac{\sin^2(x)}{\sin(x)*\left(\cos(x)+1\right)},$$ cancel out one of the sin(x)'s

$$= \lim_{x\to 0}\frac{\sin(x)}{\cos(x)+1},$$ now "put zero in" for x

$$= \frac{0}{1+1}$$

$$= 0$$

From above, $\lim_{x\to 0}\frac{\cos(x)-1}{\sin(x)} = 0$

45. Find the given limit.

For this problem you must know: sin(2x) = 2*sin(x)*cos(x).

$$\lim_{x->\pi} \frac{\tan(x)}{\sin(2x)} = \lim_{x->\pi} \frac{\sin(x)}{\cos(x)} * \frac{1}{\sin(2x)},$$ put tan(x) in terms of sin() and cos()

$$= \lim_{x->\pi} \frac{\sin(x)}{\cos(x)} * \frac{1}{2\sin(x)\cos(x)},$$ apply double angle formula

$$= \lim_{x->\pi} \frac{1}{\cos(x)} * \frac{1}{2\cos(x)},$$ simplify

$$= \lim_{x->\pi} \frac{1}{2\cos^2(x)},$$ "put in π" for x

$$= \frac{1}{2*1^2}$$

= ½

$$\lim_{x->\pi} \frac{\tan(x)}{\sin(2x)} = ½$$

48. Find the given limit.

For this one you need to remember Limit Law #4 from section 2.3:

$\lim_{x\to a} [f(x) * g(x)] = \lim_{x\to a} [f(x)] * \lim_{x\to a} [g(x)]$

And Theorem 4, from this section: $\lim_{\theta\to 0} [\sin(\theta) / \theta] = 1$

$$\lim_{x\to 1}\frac{\sin(x-1)}{x^2+x-2} = \lim_{x\to 1}\frac{\sin(x-1)}{(x+2)(x-1)},$$ factor the denominator

$$= \left(\lim_{x\to 1}\frac{1}{x+2}\right)*\left(\lim_{x\to 1}\frac{\sin(x-1)}{x-1}\right),$$ apply limit law #4

$$= \left(\lim_{x\to 1}\frac{1}{x+2}\right)*(1),$$ apply $\lim_{\theta\to 0} [\sin(\theta) / \theta]$

$$= \left(\frac{1}{1+2}\right)*(1),$$ "put 1 in for x"

= 1/3

$$\lim_{x\to 1}\frac{\sin(x-1)}{x^2+x-2} = 1/3$$

Notice some software packages (and calculators) may believe you are operating in degrees (i.e. the software thinks that x → 1°. If this is the case it will return the answer to be: π/540, which is the same thing as (1/3)°. To prevent this make certain the calculator or software program is in RADIAN mode.

Section 5.7

(continuing from 3.4)

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For this section you might want to refer to the below table:

Function Given	Its Anti-Derivative
$k * x^n$ (and $n \neq -1$) *where k is a constant*	$\frac{k * x^{n+1}}{n+1}$ *where k is a constant*
$\frac{1}{x}$	ln(| x |)
k * e^x *where k is a constant*	k * e^x *where k is a constant*
k * cos(x) *where k is a constant*	k * sin(x) *where k is a constant*
k * sin(x) *where k is a constant*	-k * cos(x) *where k is a constant*
k * $\sec^2(x)$ *where k is a constant*	k * tan(x) *where k is a constant*
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1}(x)$
$\frac{1}{1+x^2}$	$\tan^{-1}(x)$

Section 5.7
11. h(x) = sin(x) – 2*cos(x)

So h(x) = f(x) + g(x), where f(x) = sin(x) and g(x) = –2 * cos(x)

H(x) = F(x) + G(x) + C

f(x) = sin(x) → F(x) = –cos(x), from looking at the table above
g(x) = –2*cos(x) → G(x) = –2*sin(x), from looking at the table above

Thus,
H(x) = F(x) + G(x) + C
= –cos(x) + (–2)*sin(x) + C
= –cos(x) – 2sin(x) + C

Do NOT forget the +C !!!

The antiderivative of h(x) = H(x) = –cos(x) – 2sin(x) + C

Section 5.7
12. f(t) = sin(t) – 2*$t^{1/2}$.

Notice we rewrote $2\sqrt{t}$ as $t^{1/2}$. And as with problem 11 we see we can break the given function into the addition of two separate functions:

f(t) = g(t) + h(t), where g(t) = sin(t) and h(t) = –2*$t^{1/2}$.
F(t) = G(t) + H(t) + C

g(t) = sin(t) → G(t) = -cos(t)
h(t) = –2*$t^{1/2}$ → H(t) = (–2 * $t^{1/2+1}$) / (½ + 1) = (–2 * $t^{3/2}$) / (3/2) = (–4/3)*$t^{3/2}$.

Thus,
F(t) = G(t) + H(t) + C
F(t) = –cos(t) + (–4/3)*$t^{3/2}$ + C
= –cost(t) – (4/3)*$t^{3/2}$ + C

The antiderivative of f(t) = F(t) = –cost(t) – (4/3) * $t^{3/2}$ + C.