Section 3.7
Solutions and Hints

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for the book:
Calculus, Early Vectors
by James Stewart.

Velocity is a vector. Speed is a scalar. The derivative of a position vector is velocity. Unit vectors have magnitude (length) of one.

18. The vector function r(t) represents the position of a particle at time t. Find the velocity and speed at the given value of t.

$\mathbf{r}(t) = \sqrt{t^2+5}\,\mathbf{i} + t\mathbf{j}, \quad t = 2$

This gives us: $x(t) = (t^2 + 5)^{1/2}$ and $y(t) = t$
We need to find x'(t) and y'(t). We will need to apply the chain rule to find x'(t).

Finding x'(t):

Let the outer function, O(stuff), be the sqrt function. Let stuff = $t^2 + 5$

$O(stuff) = (stuff)^{1/2}$ stuff = $t^2 + 5$
$O'(stuff) = (½)*(stuff)^{-1/2}$ stuff ' = 2t

By the chain rule:

$x'(t) = O'(stuff) * stuff'$
$= (½)*(stuff)^{-1/2} * 2t,$ *put t^2+5 in for stuff.*
$= (½)*(t^2 + 5)^{-1/2} * 2t$
$= \dfrac{t}{\sqrt{t^2+5}}$

Finding y'(t):

$y(t) = t \rightarrow y'(t) = 1$

Put it together:

Velocity = $v(t) = x'(t)\mathbf{i} + y'(t)\mathbf{j} = \dfrac{t}{\sqrt{t^2+5}}\mathbf{i} + \mathbf{j}.$

$v(2) = 2/3\mathbf{i} + \mathbf{j}.$

Speed at t = 2 is $|v(2)| = [\ 4/9 + 1\ ]^{1/2} = \dfrac{\sqrt{13}}{3}$