Section 3.9
Solutions and Hints

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for the book:
Calculus, Early Vectors
by James Stewart.

14. Find the point(s) on the curve where the tangent line is horizontal or vertical.

$x(t) = \sin(2t)$ $\quad$ $y(t) = \sin(t)$
$x'(t) = 2*\cos(2t)$ $\quad$ $y'(t) = \cos(t)$

Slope = $y'(t) / x'(t) = dy/dx$
So the tangent would be vertical at $x' = 0$ and horizontal at $y' = 0$.

$2*\cos(2t) = 0 \rightarrow \cos(2t) = 0$
$\rightarrow 2t = \cos^{-1}(0)$
$\rightarrow 2t = \pi/2$
$\rightarrow t = \pi/4$

So at $t = (2k+1)*\pi/4$ the tangent would be vertical where k is any integer.
Thus the full solution we need to consider is $t = \pi/4$ AND $t = 3*\pi/4$:

$x(\pi/4) = \sin(2*\pi/4) = 0$ $\quad$ $y(\pi/4) = \sin(\pi/4) = \frac{\sqrt{2}}{2}$

$x(3*\pi/4) = \sin(2*3*\pi/4) = -1$ $\quad$ $y(\pi/4) = \sin(3*\pi/4) = \frac{\sqrt{2}}{2}$

$\cos(t) = 0 \rightarrow t = \cos^{-1}(0)$
$\rightarrow t = \pi/2$

So at $t = (2k+1)*\pi/2$ the tangent would be horizontal where k is any integer.
Thus we must consider $t = \pi/2$ and $3*\pi/2$

$x(\pi/2) = \sin(2*\pi/2) = 0$ $\quad$ $y(\pi/2) = \sin(\pi/2) = 1$
$x(3*\pi/2) = \sin(2*3*\pi/2) = 0$ $\quad$ $y(\pi/2) = \sin(3*\pi/2) = -1$

At $(0, \frac{\sqrt{2}}{2})$ and $(-1, \frac{\sqrt{2}}{2})$ the tangent is vertical.
At (0,1) and (0, -1) the tangent is horizontal.