Section 3.10
Solutions and Hints

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for the book:

Calculus, Early Vectors
by James Stewart.

6. If a snowball melts so that its surface area decreases at a rate of 1 cm^2 per minute, find the rate at which the DIAMETER decreases when the diameter is 10 cm.

Let V = volume of a sphere = $(4/3)\pi r^3$.
We are given that dV/dt = 1. We must find dr/dt so we can find dd/dt, where t = time, r = radius and d = 2r = diameter.

We being with the formula for volume and differentiate both sides:

$V = (4/3)\pi r^3 \rightarrow d/dt[\ V\ ] = d/dt[\ (4/3)\pi r^3\ ]$

$\rightarrow dV/dt = (4/3)\pi * 3 * r^2 * dr/dt$ *put 1 for dV/dt.*

$\rightarrow 1 = 4\pi * r^2 * dr/dt$ *solve for dr/dt.*

$$\rightarrow \frac{1}{4\pi * r^2} = dr/dt$$

Given d = 2r, when d = 10 then r = 5. So we put 5 in for r:

$$dr/dt = \frac{1}{4\pi * r^2} = \frac{1}{4\pi * 5^2} = \frac{1}{100\pi}$$

Now we need to find dd/dt, given that dr/dt = $\frac{1}{100\pi}$.

$d = 2r \rightarrow d/dt[\ d] = d/dr[\ 2r\ ]$

$\rightarrow dd/dt = 2*dr/dt$

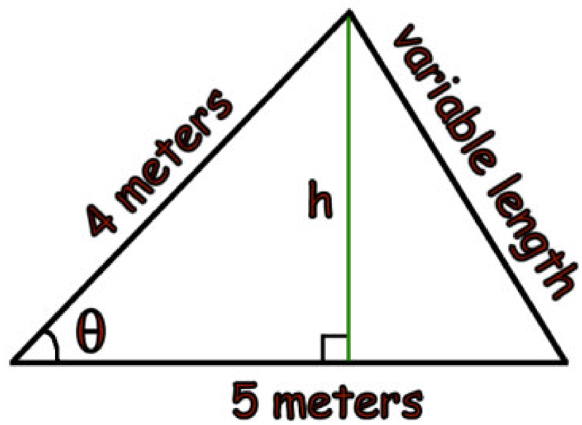
$$\rightarrow dd/dt = \frac{2}{100\pi}$$

The diameter is decreasing at a rate of dd/dt = $\frac{1}{50\pi}$ cm/sec when the diameter is 10 cm.

23. Two sides of a triangle are 4 meters and 5 meters in length. The angle between them is increasing at a rate of 0.06 radians per second. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is $\pi/3$.

As long as you set this up correctly everything is easy. The way to think is:

Area is dependent on base and height. One of those needs to be constant and the other needs to be a function of θ because we are given dθ/dt.



As we have drawn this we see the base of the main triangle will always be 5 m. We also should see that sin(θ) = h / 4 → h = 4*sin(θ). So we have successfully set stuff up so h is a function of θ.

We know: Area = A = ½*(base)*height.

We have height = h = 4*sin(θ)

So dh/dt = d/dt[4*sin(θ)]

dh/dt = 4*cos(θ)*dθ/dt, *put 0.06 in for dθ/dt.*

= 4*cos(θ)*0.06 *put π/3 in for θ.*

= 4* ½ * 0.06

= 0.12

And now we can use dh/dt to find dA/dt:

Area = ½ * (base) * height, *base is constant and = 5.*

A = ½ * 5 * h

Thus dA/dt = d/dt [(5/2) * h]

= (5/2) * dh/dt, *and we found dh/dt = 0.12.*

= 2.5 * 0.12

= 0.3 m^2 / s

When θ = π/3 the area of the triangle is changing at the rate of **0.3 m^2 / s**.