Section 3.11
Solutions and Hints

by Brent M. Dingle

for the book:
<u>Calculus, Early Vectors</u>
by James Stewart.

35. Verify the linear approximation at a = 0.

This question is open to some interpretation.
The given approximation is: $\sqrt{1+x} \cong 1 + ½*x$

<u>The easiest interpretation means put 0 in for x and show the approximation holds:</u>

$\sqrt{1+0} \cong 1 + ½ * 0$
1 = 1
Since 1 = 1 the approximation is definitely true at a = 0.

<u>It is more likely they meant for you to derive the linear approximation</u>:

Let f(x) = $\sqrt{1+x}$ → f(0) = 1
Then f '(x) = ½ * $(1 + x)^{-1/2}$ → f '(0) = ½ * $(1 + 0)^{-1/2}$ = ½

Let L(x) denote the linear approximation of f(x) at a, by definition:

L(x) = f(a) + f'(a)(x – a), *for this problem a = 0*
= f(0) + f'(0)(x – 0) *f(0) and f '(0) where found above*
= 1 + ½*x

And thus we have verified the suggested linear approximation.

36. Verify the linear approximation at a = 0.

This question is open to some interpretation.
The given approximation is: $\sin(x) \cong x$

Notice if you put zero in for x you get sin(0) = 0 and arrive at a truth, so it is possible the suggested approximation is true. However, it is most likely the author(s) intended for you to derive the linear approximation as follows (and thus you will have verified it):

$f(x) = \sin(x) \rightarrow f(0) = \sin(0) = 0$
$f'(x) = \cos(x) \rightarrow f'(0) = \cos(0) = 1$

By definition if L(x) is the linear approximation of f(x) at x = a then:

$L(x) = f(a) + f'(a)(x - a)$, *in this case a = 0.*
$= f(0) + f'(0)(x - 0)$, *f(0) and f '(0) where derived above.*
$= 0 + 1x$
$= x$

And thus we have verified that the linear approximation of sin(x) near x = 0 is indeed just x (i.e. $\sin(x) \cong x$, at x near 0).

42. Determine the values of x for which the linear approximation is accurate to within 0.1.

The given approximation is: $\sin(x) \cong x$.

Accurate to within 0.1 means: $| \sin(x) - x | < 0.1$
Or rather, $-0.1 < \sin(x) - x < 0.1$

By the book's method you simply graph three functions:

$y1 = \sin(x) - 0.1$
$y2 = x$
$y3 = \sin(x) + 0.1$

Then zoom in near $x = 0$ and see where the difference between the graphs is < 0.1. (Look for where the graphs intersect).

It will work, but <u>seems</u> pretty dumb.

While there are slightly better ways, the only other one you are likely to be able to use is making a table (which is what was done before graphing calculators =)

sin(x)	**x**	**sin(x) – x**
0.7833269096	0.9	-0.1166730904
0.7512804051	0.85	-0.0987195949
0.7173560909	0.8	-0.0826439091
0	0	0
-0. 7512804051	-0.85	0.0987195949
-0.7833269096	-0.9	0.1166730904

So we would likely conclude that for the range of **x in –0.85 to 0.85**, the approximation $\sin(x) \cong x$ is accurate to within 0.1.