Section 3.12
Solutions and Hints

by Brent M. Dingle

for the book:
Calculus, Early Vectors
by James Stewart.

10. Use Newton's method to approximate the root of $x^4 + x^3 - 22x^2 - 2x + 41 = 0$ in the interval [1, 2].

$f(x) = x^4 + x^3 - 22x^2 - 2x + 41$
$f\,'(x) = 4x^3 + 3x^2 - 44x - 2$

For no real reason we will let $x_1 = 1$.

By definition $x_{n+1} = x_n - \dfrac{f(x_n)}{f\,'(x_n)} = x_n - \dfrac{x_n^4 + x_n^3 - 22x_n^2 - 2x + 41}{4x_n^3 + 3x_n^2 - 44x_n - 2}$

Which gives the following table:

n	x_n	$f(x_n)$	$f\,'(x_n)$
1	1	19	-39
2	58/39	-5670227 / 2313441	-2826194 / 59319
3	1.435735589	-0.0122348302	-47.15021813
4	1.435476103	-0.0000003587	-47.14745336
5	1.435476095	-0.000000000001	-----

From iteration 4 to 5 the first 6 decimal places remain unchanged, so we "know" we are accurate to that many decimal places.

Thus by Newton's method the given function has a root at 1.435476.