Section 4.1
Solutions and Hints

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for the book:
<u>Calculus, Early Vectors</u>
by James Stewart.

18. Find the given limit.

$$\lim_{x->-\infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}$$

We know that $\lim_{x\text{-->}-\infty} e^x = 0$ and $\lim_{x\text{-->}\infty} e^x = \infty$.
So we need to somehow "get rid of" the exponents 3x an –3x.
To do this we will divide everything by e^{-3x}.

$$\lim_{x->-\infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}} = \lim_{x->-\infty} \frac{\left(\frac{e^{3x}}{e^{-3x}} - \frac{e^{-3x}}{e^{-3x}}\right)}{\left(\frac{e^{3x}}{e^{-3x}} + \frac{e^{-3x}}{e^{-3x}}\right)}$$

$$= \lim_{x->-\infty} \frac{e^{6x} - 1}{e^{6x} + 1}$$

$$= \frac{\lim_{x->-\infty} e^{6x} - \lim_{x->-\infty} 1}{\lim_{x->-\infty} e^{6x} + \lim_{x->-\infty} 1}, \qquad \textit{apply } \lim_{x\text{-->}-\infty} e^x = 0$$

$$= \frac{0-1}{0+1}$$

$$= -1$$

$$\lim_{x->-\infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}} = -1$$

Notice if we had divided everything by e^{3x} we would have ended at $-\infty/\infty$ which does not help, thus we had to try something that resulted in a more definitive result.

28. Differentiate f(x) = xe^{x^2}

Given f(x) = $x * e^{x^2}$, we see f(x) can be written as:

f(x) = g(x)*h(x), where g(x) = x and h(x) = e^{x^2}

Easily g(x) = x → g'(x) = 1

However we must use the Chain Rule to find h'(x):

Let the outer function be O(stuff) = e^{stuff}, with stuff = x^2.

O(stuff) = e^{stuff} stuff = x^2

O'(stuff) = e^{stuff} stuff ' = 2x

h'(x) = O'(stuff) * stuff '

= e^{stuff} * 2x *put x^2 in for stuff.*

= e^{x^2} * 2x

Thus h'(x) = 2x* e^{x^2}

And by the Product Rule, f '(x) = g'(x)*h(x) + g(x)*h'(x)

f'(x) = 1* e^{x^2} + x * (2x* e^{x^2})

= e^{x^2} + $2x^2$* e^{x^2}

f '(x) = e^{x^2} + $2x^2$* e^{x^2}

31. Differentiate the given function.

Given: h(t) = $\sqrt{1-e^t}$

We see that the Chain Rule can be applied if we let the outer function, O(stuff) be the square root of stuff and stuff = 1 – e^t.

O(stuff) = $(\text{stuff})^{1/2}$ stuff = 1 – e^t

O'(stuff) = ½*$(\text{stuff})^{-1/2}$ stuff ' = –e^t

h'(t) = O'(stuff) * stuff '

= ½*$(\text{stuff})^{-1/2}$ * (–e^t), *put 1 – e^t in for stuff*

=1/2*$(1-e^t)^{-1/2}$ * (–e^t)

And h'(t) = $\dfrac{-e^t}{2\sqrt{1-e^t}}$

47. Show that the function $y = e^{2x} + e^{-3x}$ satisfies the differential equation: $y'' + y' - 6y = 0$.

To show this all you need to do is find y' and y''. Put them (and y) into the given differential equation and simplify it (correctly) to 0.

Notice to find the derivative of $c*e^{k*x}$ we need to apply the chain rule with the outer function, $O(stuff) = c*e^{stuff}$ and stuff = k*x which gives:

$O(stuff) = c*e^{stuff}$ stuff = k*x

$O'(stuff) = c*e^{stuff}$ stuff ' = k

Thus $d/dx(c*e^{k*x}) = O'(stuff) * stuff'$
$= c*e^{stuff} * k$
$= c*k*e^{k*x}$.

From that we know:

$d/dx(e^{2x}) = 2*(e^{2x})$,   *c = 1 and k = 2.*

and

$d/dx(e^{-3x}) = -3* (e^{-3x})$,   *c = 1 and k = -3.*

So $\mathbf{y' = 2e^{2x} - 3e^{-3x}}$

And $\mathbf{y'' = 4e^{2x} + 9e^{-3x}}$ *c = 2 and k = 2 and then c = -3 and k = -3.*

Now we put everything into the given differential equation:

$$y'' + y' - 6y = (4e^{2x} + 9e^{-3x}) + (2e^{2x} - 3e^{-3x}) - 6*(e^{2x} + e^{-3x})$$
$$= 6e^{2x} + 6e^{-3x} - 6e^{2x} - 6e^{-3x}$$
$$= 0$$

And we have shown $y'' + y' - 6y = 0$. So we are done.