Section 4.2
Solutions and Hints

by Brent M. Dingle

for the book:
<u>Calculus, Early Vectors</u>
by James Stewart.

For this section you need to know:

<u>How to find the inverse of a one-to-one function f(x)</u>:

You are given f(x) = blah, blah, blah

1. Write y = f(x)
2. Solve the equation for x in terms of y (this is not always possible).
3. Switch the x and y.
4. The result is $y = f^{-1}(x)$

IMPORTANT: Notice that $f(x) = x^2$ is NOT a one-to-one function → f(2) = f(-2) = 4. However if you restrict its domain to say x ≥ 0 then you can 'pretend' it is.

<u>Example</u>:

Given $f(x) = x^5 - 3$ find $f^{-1}(x)$

1. Write y = f(x): $y = x^5 - 3$

2. Solve for x: $y + 3 = x^2$
$x^5 = y + 3$
$x = \sqrt[5]{y+3}$

3. Switch x and y: $y = \sqrt[5]{x+3}$

4. Result is $y = f^{-1}(x)$: $f^{-1}(x) = \sqrt[5]{x+3}$

So the inverse of $x^5 - 3$ is $\sqrt[5]{x+3}$

You should also memorize Theorem 7 (illustrated in problem 26 below).

26. Find g'(a), where g is the inverse function of the given function.

Given: $f(x) = x^5 - x^3 + 2x$, and $a = 2$

Recall that Theorem 7 of this section says:
If f is one-to-one and differentiable with inverse g and $f'(g(a)) \neq 0$
then

$$g'(a) = \frac{1}{f'(g(a))}$$

Graph f(x) and make sure it passes the horizontal line test – so it is one-to-one. Because it is a polynomial we know it is continuous and differentiable (specifically at x = 2). Thus Theorem 7 applies:

We need to find g(2). Notice that $g(2) = f^{-1}(2)$.
Recall $f(f^{-1}(c)) = f(g(c)) = c$
So we start with:

$g(2) = c$ *take f( ) of both sides*
$f(g(2)) = f(c)$ *f(g(2)) = 2*
$2 = f(c)$

Thus to find g(2) we can set f(c) = 2 and solve for c.

$f(c) = c^5 - c^3 + 2c = 2$
$c^5 - c^3 + 2c - 2 = 0$ *There is no real good way to solve this. For the most part the answers are usually small such as 0, 1, 2 or 3. So sadly 'guessing' turns out to be the fastest method. This actually factors:*
$(c - 1)*(c^4 + c^3 + 2) = 0$

So c = 1 is the only real solution. Thus f(1) = 2 → 1 = g(2).

And $f(x) = x^5 - x^3 + 2x$ → $f'(x) = 5x^4 - 3x^2 + 2$

Thus $f'(g(2)) = f'(1) = 5*1 - 3*1 + 2 = 5 - 3 + 2 = 4$

$$g'(2) = \frac{1}{f'(g(2))} = \frac{1}{4}$$

And we conclude g'(2) = ¼.

31. Suppose g is the inverse function of f and f(4) = 5, f'(4) = 2/3. Find g'(5).

This is just an application of Theorem 7 of this section, which basically says:

$$g'(a) = \frac{1}{f'(g(a))}$$

Here because	f(4) = 5	*we take g() of both sides*
we know	g(f(4)) = g(5)	*as g and f are inverse g(f(4)) = 4*
and thus	4 = g(5)	

This is not surprising as the only other thing we are given is f '(4) = 2/3.

So f '(g(5)) = f '(4) = 2/3

$$\text{Thus } g'(5) = \frac{1}{(2/3)} = 3/2$$

So g'(5) = 3/2 = 1.5