Section 4.3
Solutions and Hints

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for the book:
Calculus, Early Vectors
by James Stewart.

46. Solve for x:

ln(x) + ln(x – 1) = 1 → ln(x * (x – 1)) = 1

→ ln(x^2 – x) = 1

→ $e^{\ln(x^2 - x)} = e^1$

→ $x^2 - x = e$

→ $x^2 - x - e = 0$, *apply the quadratic formula.*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1 - 4*1*(-e)}}{2}$$

$$= \frac{1 \pm \sqrt{1+4e}}{2}$$

$$\cong 2.22287023 \text{ or } -1.22287023$$

As we cannot take the natural log of negative numbers we ignore the second possibility and conclude $x = \frac{1+\sqrt{1+4e}}{2} \cong 2.22287023$

$$x = \frac{1+\sqrt{1+4e}}{2} \cong 2.22287023$$

51. Solve for x:

$\ln(x + 6) + \ln(x - 3) = \ln(5) + \ln(2)$ → $\ln((x+6)(x - 3)) = \ln(5*2)$
→ $\ln(x^2 + 3x - 18) = \ln(10)$
→ $e^{\ln(x^2+3x-18)} = e^{\ln(10)}$
→ $x^2 + 3x - 18 = 10$
→ $x^2 + 3x - 28 = 0$, *factor*
→ $(x + 7)(x - 4) = 0$

So x = -7 or x = 4, notice that ln(-7 + 6) = ln(-1) which is undefined so we discard the answer x = -7 and conclude x = 4.

x = 4

52. Solve for x:

$\ln\left(\frac{x-2}{x-1}\right) = 1 + \ln\left(\frac{x-3}{x-1}\right)$ → $\ln\left(\frac{x-2}{x-1}\right) - \ln\left(\frac{x-3}{x-1}\right) = 1$

→ $\ln\frac{\left(\frac{x-2}{x-1}\right)}{\left(\frac{x-3}{x-1}\right)} = 1$

→ $\ln\left(\left(\frac{x-2}{x-1}\right)*\left(\frac{x-1}{x-3}\right)\right) = 1$

→ $\ln\left(\frac{x-2}{x-3}\right) = 1$

→ $e^{\ln\left(\frac{x-2}{x-3}\right)} = e^1$

→ $\left(\frac{x-2}{x-3}\right) = e$

→ $x - 2 = e(x - 3)$
→ $x - 2 = ex - 3e$
→ $x - ex = 2 - 3e$
→ $x(1 - e) = 2 - 3e$
→ $x = \frac{2-3e}{1-e}$

$x = \frac{2-3e}{1-e}$

70. Find the limit:

$\lim_{x \to \infty} [\ \ln(2 + x) - \ln(1 + x)\ ]$, *notice as-is we would get ∞ - ∞, so simplify*

$= \lim_{x \to \infty} \ln\left(\frac{2+x}{1+x}\right)$, *not much improvement get ln(∞ / ∞), so*

divide all by x (or mult by (1/x) / (1/x))

$= \lim_{x \to \infty} \ln\left(\frac{2+x}{1+x} * \frac{(1/x)}{(1/x)}\right)$

$= \lim_{x \to \infty} \ln\left(\frac{\left(\frac{2}{x} + \frac{x}{x}\right)}{\left(\frac{1}{x} + \frac{x}{x}\right)}\right)$, *now we can "put ∞ in" for x*

$= \lim_{x \to \infty} \ln\frac{0+1}{0+1}$

$= \lim_{x \to \infty} \ln(\ 1\ )$

$= \lim_{x \to \infty} 0$

$= 0$

$\lim_{x \to \infty} [\ \ln(2 + x) - \ln(1 + x)\ ] = 0$