Section 4.4
Solutions and Hints

by Brent M. Dingle

for the book:
Calculus, Early Vectors
by James Stewart.

28. Differentiate $G(x) = 5^{\tan(x)}$.

For this one we will need to apply the chain rule (as almost always now).
Let the outer function $O(\text{stuff}) = 5^{\text{stuff}}$. Let stuff = tan(x).

$O(\text{stuff}) = 5^{\text{stuff}}$	stuff = tan(x)
$O'(\text{stuff}) = (5^{\text{stuff}})^* \ln(5)$	$\text{stuff}' = \sec^2(x)$

Then by the Chain Rule:

$G'(x) = O'(\text{stuff}) * \text{stuff}'$

$= [(5^{\text{stuff}})^* \ln(5)] * \sec^2(x)$, *put tan(x) in for stuff.*

$= (5^{\tan(x)})^* \ln(5) * \sec^2(x)$

$G'(x) = = (5^{\tan(x)})^* \ln(5) * \sec^2(x)$

34. Differentiate $y = x^x$.

This one has shown up on tests in the past.

Usually when you have x in the base AND in the exponent you will need to either take the natural log of both sides or rewrite the right hand side in terms using $e^{something}$. Example 9 in the book illustrates both techniques well.

For this problem we will opt to take the ln() of both sides:

$y = x^x$ → $\ln(y) = \ln(x^x)$, *This allows us to get the x out of the exponent.*

→ $\ln(y) = x * \ln(x)$, *But this next part is a little confusing.*

Now we must differentiate both sides with respect to x, but remember we are trying to find y' (or rather dy/dx).

$\ln(y) = x * \ln(x)$ → $d/dx[\ \ln(y)\ ] = d/dx[\ x * \ln(x)\ ]$

<u>We will first find d/dx[ln(y)]</u>:

Remember y is a function of x. So the chain rule gives:

O(stuff) = ln(stuff) stuff = y

O'(stuff) = 1 / stuff stuff ' = y'

d/dx[ln(y)] = (1 / stuff) * stuff ', *put y in for stuff.*

= (1 / y) * y'

= y' / y, *recall that* $y = x^x$.

= **y' / (x^x)**

<u>And now we find d/dx[x * ln(x)]</u>:

For this we just apply the Product Rule, with g(x) = x, h(x) = ln(x)

g(x) = x h(x) = ln(x)

g'(x) = 1 h'(x) = 1/x

d/dx[x*ln(x)] = g'(x)*h(x) + g(x)*h'(x)

= 1*ln(x) + x*(1/x)

= **ln(x) + 1**

<u>Putting stuff together</u>:

$\ln(y) = x * \ln(x)$ → $d/dx[\ \ln(y)\ ] = d/dx[\ x * \ln(x)\ ]$

→ $y' / (x^x) = \ln(x) + 1$, *solve for y'.*

→ $y' = (\ \ln(x) + 1\ ) * x^x$

$y' = (\ \ln(x) + 1) * x^x$.

35. Differentiate $y = x^{\sin(x)}$.

As with problem 24, we have x in both the base and the exponent, so again we will opt to take the ln() of both sides:

$y = x^{\sin(x)}$ → $\ln(y) = \ln(x^{\sin (x)})$

→ $\ln(y) = \sin(x) * \ln(x)$

Now we must differentiate both sides with respect to x, but remember we are trying to find y' (or rather dy/dx).

$\ln(y) = \sin(x) * \ln(x)$ → $d/dx[\ln(y)] = d/dx[\sin(x) * \ln(x)]$

<u>We will first find d/dx[ln(y)]</u>:

Remember y is a function of x. So the chain rule gives:

O(stuff) = ln(stuff)	stuff = y
O'(stuff) = 1 / stuff	stuff ' = y'

d/dx[ln(y)] = (1 / stuff) * stuff ', *put y in for stuff.*

= (1 / y) * y'

= y' / y, *recall that* $y = x^{\sin(x)}$.

= **y' / ($x^{\sin(x)}$)**

<u>And now we find d/dx[sin(x) * ln(x)]</u>:

For this we just apply the Product Rule, with g(x) = sin(x), h(x) = ln(x)

g(x) = sin(x)	h(x) = ln(x)
g'(x) = cos(x)	h'(x) = 1/x

d/dx[x*ln(x)] = g'(x)*h(x) + g(x)*h'(x)

= cos(x)*ln(x) + sin(x)*(1/x)

= **cos(x) * ln(x) + sin(x)/x**

<u>Putting stuff together</u>:

ln(y) = sin(x)*ln(x) → d/dx[ln(y)] = d/dx[sin(x) * ln(x)]

→ y' / ($x^{\sin(x)}$) = cos(x)*ln(x) + sin(x)/x, *solve for y'.*

→ y' = [cos(x)*ln(x) + sin(x)/x] * $x^{\sin(x)}$

$y' = [\cos(x)*\ln(x) + \sin(x)/x] * x^{\sin(x)}$