Section 4.5
Solutions and Hints

by Brent M. Dingle

for the book:
<u>Calculus, Early Vectors</u>
by James Stewart.

2. A bacteria culture starts with 4000 bacteria and the populations triples every half-hour.
2a. Find an expression for the number of bacteria after *t* hours.

This problem is a direct application of: $y(t) = y(0)*e^{k*t}$.

We are given y(0) = 4000 and since the population doubles in half an hour we also know y(½) = 8000. This will allow us to solve for k:

$y(½) = y(0)*e^{k * ½}$ → $8000 = 4000 * e^{k * ½}$
→ $2 = e^{k * ½}$
→ $\ln(2) = \ln(e^{k * ½})$
→ $\ln(2) = k * ½$
→ $2*\ln(2) = k$

So our equation is: $y(t) = 4000 * e^{2*\ln(2)*t}$

2b. Find the number of bacteria after 20 minutes.

Notice 20 minutes = 1/3 hour. So put 1/3 into the equation from part (a):
$y(1/3) = 4000*e^{2*\ln(2)*1/3} = 4000*2^{2/3} \cong 6349$ (round down to nearest whole bacteria)

$y(1/3) \cong 6349$ bacteria.

2c. When will the population reach 20,000 ?

This question is asking at what value of t does y(t) = 20000.
So set our equation from part (a) = 20000 and solve for t.

$20000 = 4000 * e^{2*\ln(2)*t}$

→ $5 = e^{2*\ln(2)*t}$
→ $\ln(5) = \ln(e^{2*\ln(2)*t})$
→ $\ln(5) = 2 * \ln(2) * t$
→ $\frac{\ln(5)}{2*\ln(2)} = t$
→ $t \cong 1.16$ hours

$$t = \frac{\ln(5)}{2*\ln(2)} \cong 1.16 \text{ hours.}$$

8. Polonium-210 has a half-life of 140 days.

8a. If a sample has a mass of 200 mg, find a formula for the mass that remains after *t* days.

Start with: $y(t) = y(0)*e^{k*t}$.
We know y(0) = 200 mg and y(140) = 100 mg, use that to find k:

$y(140) = y(0)*e^{k*140}$

→ $100 = 200*e^{140*k}$
→ $½ = e^{140*k}$
→ $\ln(½) = \ln(e^{140*k})$
→ $\ln(½) = 140 * k$
→ $\ln(½) / 140 = k$

Putting everything into $y(t) = y(0)*e^{k*t}$ we get:

$y(t) = y(0)*e^{k*t}$.

→ $y(t) = 200*e^{[\ln(1/2) * t]/ 140}$, recall $\ln(1/2) = \ln(2^{-1}) = -\ln(2)$
→ $y(t) = 200*e^{[-\ln(2) * t]/ 140}$

$$y(t) = 200*e^{[-\ln(2) * t]/ 140}$$

8b. Find the mass after 100 days.

For this all that needs done is to put 100 into the equation derived in part (a):
$y(100) = 200*e^{[\ln(1/2) * 100]/140} \cong 121.901$ mg

$y(100) \cong 121.901$ mg

8c. When will the mass be reduced to 10 mg?

Here we set $y(t) = 200*e^{[-\ln(2) * t]/140} = 10$ and solve for t:

$10 = 200*e^{[-\ln(2) * t]/140}$

$\rightarrow 1/20 = e^{[-\ln(2) * t]/140}$

$\rightarrow \ln\left(\frac{1}{20}\right) = \ln(e^{[-\ln(2) * t]/140})$

$\rightarrow \ln\left(\frac{1}{20}\right) = \left(\frac{-\ln(2)}{140}\right) * t$

$\rightarrow 140 * (-\ln(20)) = -\ln(2) * t$

$\rightarrow \frac{140 * \ln(20)}{\ln(2)} = t$

$\rightarrow t \cong 605.07$ years

After $t = \frac{140 * \ln(20)}{\ln(2)} \cong$ **605.07** years only 10 mg will remain.

8d. Sketch the graph of the mass function.

This is left for you and/or your calculator or Maple.

20. How long will it take an investment to double in value if the interest rate is 6% compounded continuously?

Notice the formula for interest compounded annually, semiannually, quarterly, etc is NOT the same as for continuously. Be absolutely certain you know both equations.

For interest continuously compounded we have:

$A(t) = A_0 * e^{i*t}$ where *t* is measured in years

For this problem it seems we are only given i = 0.06.
However all we need to know is the time it would take to double ANY investment, so we can set A_0 = whatever we want. So let $A_0 = 1$.

This give us: $A(t) = 1 * e^{0.06 * t}$

We need to find *t* such that A_0 doubles → goes to 2.
So set A(t) = 2 and solve for t:

$A(t) = 2 = 1 * e^{0.06 * t}$ → $2 = e^{0.06 * t}$
→ $\ln(2) = \ln(e^{0.06 * t})$
→ $\ln(2) = 0.06 * t$
→ $\ln(2) / 0.06 = t$
→ $t \cong 11.5525$ years.

It will take, $t = \ln(2) / 0.06 \cong 11.5535$ years for any investment to double.