Section 4.6
Solutions and Hints

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for the book:
Calculus, Early Vectors
by James Stewart.

For this section (and for any test) you will likely find the below table useful:

$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\left(\csc^{-1}(x)\right) = \frac{-1}{x\sqrt{x^2-1}}$
$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\left(\sec^{-1}(x)\right) = \frac{1}{x\sqrt{x^2-1}}$
$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$	$\frac{d}{dx}\left(\cot^{-1}(x)\right) = \frac{-1}{1+x^2}$

30. Find the derivative of f(x) = sin $^{-1}$(2x – 1).

This is a chain rule problem.

O(stuff) = sin $^{-1}$(stuff)   stuff = 2x – 1

O'(stuff) = $\frac{1}{\sqrt{1-stuff^2}}$ stuff ' = 2

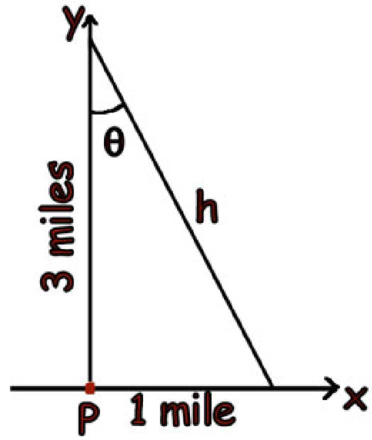
f '(x) = O'(stuff) * stuff '

$= \frac{1}{\sqrt{1-stuff^2}} * 2,$ *put 2x-1 in for stuff*

$= \frac{2}{\sqrt{1-(2x-1)^2}},$ $1 - (2x-1)^2 = 4x - 4x^2$, so a 2 is in the denominator.

f '(x) = $\frac{1}{\sqrt{x-x^2}}$

66. A lighthouse is on a small island 3 km away from the nearest point P on a straight shoreline and its light makes four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 kilometer from P ?



We are given that $d\theta/dt = 4*2\pi$ (4 revolutions per minute)
and this rate is constant, so it doesn't matter what θ or t is.

We need to find dx/dt when x = 1.
Notice that for this problem y remains constant at 3.

From the above picture we see that $\tan(\theta) = x / y$, or rather $\theta = \tan^{-1}(x/y)$.
As y is constant with respect to t we can put 3 in for it so:

$$\theta = \tan^{-1}(x / 3)$$

If we differentiate both sides with respect to t we get:

$$\theta = \tan^{-1}(x / 3) \quad \rightarrow d\theta / dt = d/dt[\tan^{-1}(x / 3)]$$

$$\rightarrow d\theta / dt = \frac{1}{1+\left(\frac{x}{3}\right)^2} * \frac{1}{3} * dx/dt$$

$$\rightarrow d\theta / dt = \frac{3}{x^2+9} * dx/dt$$

And we can now put 8π in for dθ/dt and 1 in for x and solve for dx/dt:

$$d\theta / dt = \frac{3}{x^2+9} * dx/dt \quad \rightarrow 8\pi = 3 / (1 + 9) * dx/dt$$

$$\rightarrow 8\pi = (3/10) * dx/dt$$

$$\rightarrow (80 / 3) * \pi = dx/dt$$

The light is moving $(80 / 3)*\pi \cong$ **84 km / minute** at the given instant.