Section 5.2
Solutions and Hints

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for the book:
Calculus, Early Vectors
by James Stewart.

42. Find the absolute max. and min. values of f() on the given interval.

To find the absolute minimums and maximums in [a,b]:

Basically you take the derivative, find the x values where f '(x) = 0 or D.N.E. Put those x values back into f() and compare them with f(a) and f(b). Pick the (two) values of f() which are greatest and least of all.

We are given: $f(x) = \frac{x}{x+1}$, on [1, 2].

We could use the quotient rule to find the derivative. Or we might rewrite the function and apply the product rule:

$$f(x) = \frac{x}{x+1} = x * (x+1)^{-1} = g(x) * h(x)$$

Thus

$g(x) = x \qquad h(x) = (x+1)^{-1}$

$g'(x) = 1 \qquad h'(x) = -(x+1)^{-2}$

$$f'(x) = 1*(x+1)^{-1} + x*-(x+1)^{-2}$$
$$= (x+1)^{-1} - x*(x+1)^{-2}$$
$$= \frac{1}{x+1} - \frac{x}{(x+1)^2},$$
common denominator is $(x + 1)^2$.
$$= \frac{x+1-x}{(x+1)^2}$$
$$= \frac{1}{(x+1)^2},$$
notice this is undefined for x = -1.

So the only critical point is at x = -1, where f '(x) does not exist, and –1 is not in the interval [1, 2]. So the only possible min and max is at x = 1 or x = 2.
f(1) = ½ and f(2) = 2/3.

The maximum of f(x) on [1, 2] occurs at f(1) = ½.
The minimum of f(x) on [1, 2] occurs at f(2) = 2/3.

45. Find the absolute max. and min. values of f() on the given interval.

Given $f(x) = x * e^{-x}$, on [0, 2]

Apply the Product Rule to find f '(x):

$g(x) = x$ $h(x) = e^{-x}$

$g'(x) = 1$ $h'(x) = -e^{-x}$

$f'(x) = x * (-e^{-x}) + 1*e^{-x}$

$= -xe^{-x} + e^{-x}$, *This is defined everywhere.*

Set f '(x) = 0 and solve for x:

$-xe^{-x} + e^{-x} = 0 \rightarrow e^{-x} * (-x + 1) = 0$

So $e^{-x} = 0$ or $1 - x = 0$

$e^{-x} = 0 \rightarrow \ln(e^{-x}) = \ln(0)$

$\rightarrow -x = 1$

$\rightarrow x = -1$

OR

$1 - x = 0 \rightarrow 1 = x$

So the critical values of f(x) occur at x = –1 and x = 1.
–1 is NOT in [0, 2] so we ignore that one.

That leaves the possible minimums and maximums occurring at: 0, 1, or 2

$f(0) = 0$

$f(1) = e^{-1} \cong 0.3678794412$

$f(2) = 2\,e^{-2} \cong 0.2706705665$

f(x) has an absolute maximum of $e^{-1} \cong 0.3679$ at x = 1, on the interval [0, 2].
f(x) has an absolute minimum of 0 at x = 0, on the interval [0, 2].