

Section 6.1

Solutions and Hints

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for the book:
Calculus, Early Vectors
by James Stewart.

43. Find the given limit.

$$\begin{aligned}\lim_{n \rightarrow \infty} \left(\sum_{i=1}^n \frac{1}{n} * \left(\frac{i}{n} \right)^2 \right) &= \lim_{n \rightarrow \infty} \left(\sum_{i=1}^n \frac{i^2}{n^3} \right) \\ &= \lim_{n \rightarrow \infty} \left(\frac{1}{n^3} * \sum_{i=1}^n i^2 \right) \\ &= \lim_{n \rightarrow \infty} \left(\frac{1}{n^3} * \frac{n(n+1)(2n+1)}{6} \right) \\ &= \lim_{n \rightarrow \infty} \left(\frac{n}{n} * \frac{n+1}{n} * \frac{2n+1}{6n} \right) \\ &= \lim_{n \rightarrow \infty} \left(1 * \left(\frac{1}{1} + \frac{1}{n} \right) * \left(\frac{2}{6} + \frac{1}{6n} \right) \right), \text{ put } \infty \text{ in for } n \\ &= 1 * (1 + 0) * \left(\frac{1}{3} + 0 \right) \\ &= \frac{1}{3}\end{aligned}$$

$$\lim_{n \rightarrow \infty} \left(\sum_{i=1}^n \frac{1}{n} * \left(\frac{i}{n} \right)^2 \right) = \frac{1}{3}$$