Section 6.2
Solutions and Hints

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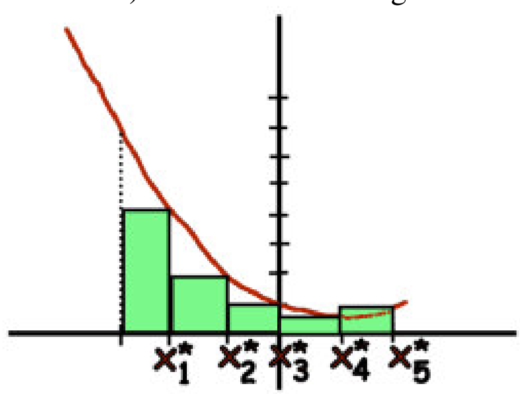
for the book:

Calculus, Early Vectors
by James Stewart.

There are a variety of programs for the TI-83 calculators that will do these problems for you. HOWEVER it is often required that you be able to set them up correctly, if for no other reason than so you can enter them into the calculator correctly.

15. Use $A = \lim_{\|P\| \to 0} \Sigma f(x_i^*)\Delta x_i$ to find the area under the given curve from a to b. Use equal subintervals and take x_i^* to be the right endpoint of the i^{th} subinterval. Sketch the region.

You are given: $y = 2x^2 - 4x + 5$ and $a = -3$ and $b = 2$.
The picture (for 5 intervals) would be something like:



$\| P \| = \Delta x_1 = \Delta x_2 = \dots = \Delta x_n = \dfrac{b-a}{n} = \dfrac{2-(-3)}{n} = \dfrac{5}{n}$

So as $\| P \| = \dfrac{5}{n}$, if $\| P \| \to 0$ then $n \to \infty$

Thus $\lim_{\| P \| \to 0}$ (stuff) is the same as $\lim_{n \to \infty}$ (stuff)

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15. continued

Because we are taking x_i^* to be the right endpoint of the intervals we have:

$$x_i^* = x_i = -3 + \frac{5}{n} * i$$

Now we can plug stuff into the given equation:

Area $= \lim_{||P||\to 0} \Sigma f(x_i^*)\Delta x_i$

$$= \lim_{n\to\infty} \sum_{i=1}^{n}\left(2*\left(-3+\frac{5i}{n}\right)^2 - 4*\left(-3+\frac{5i}{n}\right)+5\right)*\frac{5}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n}\left(2*\left(\frac{25i^2}{n^2}-\frac{30i}{n}+9\right)+12-\frac{20i}{n}+5\right)*\frac{5}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n}\left(\frac{50i^2}{n^2}-\frac{60i}{n}+18+17-\frac{20i}{n}\right)*\frac{5}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n}\left(\frac{50i^2}{n^2}-\frac{80i}{n}+35\right)*\frac{5}{n}$$

$$= \lim_{n\to\infty} \left[\sum_{i=1}^{n}\left(\frac{250i^2}{n^3}-\frac{400i}{n^2}+\frac{175}{n}\right)\right]$$

$$= \lim_{n\to\infty} \left[\sum_{i=1}^{n}\frac{250i^2}{n^3}-\sum_{i=1}^{n}\frac{400i}{n^2}+\sum_{i=1}^{n}\frac{175}{n}\right]$$

$$= \lim_{n\to\infty} \left[\frac{250}{n^3}\sum_{i=1}^{n}i^2-\frac{400}{n^2}\sum_{i=1}^{n}i+\frac{175}{n}\sum_{i=1}^{n}1\right]$$

$$= \lim_{n\to\infty} \left[\left(\frac{250}{n^3}*\frac{n(n+1)(2n+1)}{6}\right)-\left(\frac{400}{n^2}*\frac{n(n+1)}{2}\right)+\left(\frac{175}{n}*n\right)\right]$$

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15. continued again

$$= \lim_{n \to \infty} \left[\left(\frac{250}{n^3} * \frac{n(n+1)(2n+1)}{6} \right) - \left(\frac{400}{n^2} * \frac{n(n+1)}{2} \right) + \left(\frac{175}{n} * n \right) \right]$$

$$= \lim_{n \to \infty} \left[\left(\frac{n}{n} * \frac{n+1}{n} * \frac{2n+1}{6n} * 250 \right) - \left(\frac{n}{n} * \frac{n+1}{2n} * 400 \right) + (175) \right]$$

$$= \lim_{n \to \infty} \left[\left(\frac{n}{n} * \left(\frac{n}{n} + \frac{1}{n} \right) * \left(\frac{2n}{6n} + \frac{1}{n} \right) * 250 \right) - \left(\frac{n}{n} * \left(\frac{n}{2n} + \frac{1}{2n} \right) * 400 \right) + (175) \right]$$

$$= \lim_{n \to \infty} \left[\left(1 * \left(1 + \frac{1}{n} \right) * \left(\frac{2}{6} + \frac{1}{n} \right) * 250 \right) - \left(1 * \left(\frac{1}{2} + \frac{1}{2n} \right) * 400 \right) + (175) \right]$$

"put ∞ in for n"

$$= \left(1 * (1+0) * \left(\frac{2}{6} + 0 \right) * 250 \right) - \left(1 * \left(\frac{1}{2} + 0 \right) * 400 \right) + (175)$$

$$= \left(\frac{2}{6} * 250 \right) - \left(\frac{1}{2} * 400 \right) + (175)$$

$$= \frac{500}{6} - 200 + 175$$

$$= \frac{175}{3}$$

$$\cong 58.33333$$

The area under the curve: $y = 2x^2 - 4x + 5$ from $a = -3$ and $b = 2$ is $\frac{175}{3}$ units.