Section 6.3
Solutions and Hints

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for the book:
<u>Calculus, Early Vectors</u>
by James Stewart.

For this section you are expected to calculate all the integrals by applying the limit of summations definition named Riemann Integration. In practice you will rarely do this. But you may be asked to find the integral on a test using this method. Most often they will ask questions towards applying the Midpoint Rule, but not always. "Theorem 5" is the other major theorem. The properties you will notice are similar to those of summations.

<u>Theorem 5</u>:

If $f(x)$ is integrable on [a, b] then

$$\int_a^b f(x)dx = \lim_{n\to\infty} \left[\frac{b-a}{n}\sum_{i=1}^{n} f\left(a + i * \frac{b-a}{n}\right)\right]$$

<u>Midpoint Rule</u>:

$$\int_a^b f(x)dx \cong \sum_{i=1}^{n} f(\bar{x}_i)\Delta x = \Delta x * \left[f(\bar{x}_1) + \cdots + f(\bar{x}_n)\right]$$

where $\Delta x = \dfrac{b-a}{n}$

and $\bar{x}_i = \dfrac{x_{i-1} + x_i}{2}$ = the midpoint of [x_{i-1}, x_i]

18. Use Theorem 5 to evaluate the given interval.

The given integral is: $\int_1^5 (2+3x-x^2)dx$

Notice a = 1 and b = 5 and f(x) = 2 + 3x – x^2

$$\int_1^5 (2+3x-x^2)dx \quad = \lim_{n\to\infty} \left[\frac{5-1}{n}\sum_{i=1}^{n} f\left(1+i*\frac{5-1}{n}\right)\right]$$

$$= \lim_{n\to\infty} \left[\frac{4}{n}\sum_{i=1}^{n} 2+3*\left(1+i*\frac{4}{n}\right)-\left(1+i*\frac{4}{n}\right)^2\right]$$

$$= \lim_{n\to\infty} \left[\frac{4}{n}\sum_{i=1}^{n} 2+\left(3+\frac{12i}{n}\right)-\left(\frac{16i^2}{n^2}+\frac{8i}{n}+1\right)\right]$$

$$= \lim_{n\to\infty} \left[\frac{4}{n}\sum_{i=1}^{n} 2+3+\frac{12i}{n}-\frac{16i^2}{n^2}-\frac{8i}{n}-1\right]$$

$$= \lim_{n\to\infty} \left[\frac{4}{n}\sum_{i=1}^{n} 4+\frac{4i}{n}-\frac{16i^2}{n^2}\right]$$

$$= \lim_{n\to\infty} \left[\frac{4}{n}\sum_{i=1}^{n} 4+\frac{4}{n}\sum_{i=1}^{n}\frac{4i}{n}-\frac{4}{n}\sum_{i=1}^{n}\frac{16i^2}{n^2}\right]$$

$$= \lim_{n\to\infty} \left[\frac{4}{n}*4\sum_{i=1}^{n} 1+\frac{4}{n}*\frac{4}{n}\sum_{i=1}^{n} i-\frac{4}{n}*\frac{16}{n^2}\sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n\to\infty} \left[\frac{16}{n}*n+\frac{16}{n^2}*\frac{n(n+1)}{2}-\frac{64}{n^3}*\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty} \left[16+\left(\frac{16n}{2n}*\frac{n+1}{n}\right)-\left(\frac{64n}{6n}*\frac{n+1}{n}*\frac{2n+1}{n}\right)\right]$$

$$= \lim_{n\to\infty} \left[16+\left(8*\left(\frac{n}{n}+\frac{1}{n}\right)\right)-\left(\frac{32}{3}*\left(\frac{n}{n}+\frac{1}{n}\right)*\left(\frac{2n}{n}+\frac{1}{n}\right)\right)\right]$$

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18. continued

$$= \lim_{n \to \infty} \left[16 + \left(8 * \left(1 + \frac{1}{n}\right)\right) - \left(\frac{32}{3} * \left(1 + \frac{1}{n}\right) * \left(2 + \frac{1}{n}\right)\right)\right]$$

"put ∞ in for n"

$$= 16 + (8 * (1 + 0)) - \left(\frac{32}{3} * (1 + 0) * (2 + 0)\right)$$

$$= 16 + 8 - 64 / 3$$

$$= \frac{16}{3}$$

$$\int_1^5 (2 + 3x - x^2)\,dx = \frac{16}{3}$$