Section 6.4
Solutions and Hints

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for the book:
Calculus, Early Vectors
by James Stewart.

The below indefinite integral table may help for this section:

$\int c * f(x)\,dx = c * \int f(x)\,dx$	$\int [g(x) + f(x)]\,dx = \int g(x)\,dx + \int f(x)\,dx$
$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$	$\int \frac{1}{x}\,dx = \ln(\lvert x \rvert) + C$
$\int e^x\,dx = e^x + C$	$\int a^x\,dx = \frac{a^x}{\ln(a)} + C$
$\int \sin(x)\,dx = -\cos(x) + C$	$\int \cos(x)\,dx = \sin(x) + C$
$\int \sec^2(x)\,dx = \tan(x) + C$	$\int \csc^2(x)\,dx = -\cot(x) + C$
$\int \sec(x) * \tan(x)\,dx = \sec(x) + C$	$\int \csc(x) * \cot(x)\,dx = -\csc(x) + C$
$\int \frac{1}{x^2+1}\,dx = \tan^{-1}(x) + C$	$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x) + C$

58. Evaluate the integral.

Given: $\int_0^2 (x^2 - |x-1|)dx$

Here it is best to break this into two parts by rewriting the inside function:
Notice $(x^2 - |x-1|) = 0$ when $x = 1$

f(x) = $(x^2 - |x-1|)$ is the same as: f(x) = $\begin{cases} x^2 + (x-1) & x < 1 \\ x^2 - (x-1) & x \geq 1 \end{cases}$

So we can break the integral into two parts and add them together. Thus we get:

$$\int_0^2 (x^2 - |x-1|)dx = \int_0^1 \left(x^2 + (x-1)\right)dx + \int_1^2 \left(x^2 - (x-1)\right)dx$$

$$= \left[\frac{x^3}{3} + \frac{x^2}{2} - x\right]_0^1 + \left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_1^2$$

$$= \left[\left(\frac{1}{3} + \frac{1}{2} - 1\right) - \left(0 + 0 - 0\right)\right] + \left[\left(\frac{8}{3} - \frac{4}{2} + 2\right) - \left(\frac{1}{3} - \frac{1}{2} + 1\right)\right]$$

$$= \left[\frac{-1}{6}\right] + \left[\frac{8}{3} - \frac{5}{6}\right]$$

$$= \frac{5}{3}$$

$$\int_0^2 (x^2 - |x-1|)dx = \frac{5}{3}$$

77. The velocity function in m/s is given for a particle moving along a curve. Find (a) the displacement and (b) the distance traveled by the particle during the given time interval.

Displacement is how far the particle is from where it started moving.
Distance traveled is how far the particle moved to get to where it ends.

This means the particle does NOT have to move in a straight line. For example to get to San Antonio from College Station, you might go through Austin OR you might go to Houston and then to San Antonio. Obviously the distance traveled would not be the same, but your displacement would be.

We are given:
$v(t) = 3t - 5, \quad 0 \le t \le 3$

Finding the Displacement:

Recall that velocity is the derivative of position thus the integral (antiderivative) of velocity is position, specifically: $s(t) = \int v(t)\, dt$.
So

$$s(t) = \int (3t - 5)dt = (3/2)*t^2 - 5t + C$$

The displacement of the particle is just $| s(3) - s(0) | = \left| \int_0^3 s(t)dt \right|$

$$s(3) = \frac{3}{2}*3^2 - 5*3 + C \qquad = \frac{-3}{2} + C$$

$$s(0) = \frac{3}{2}*0^2 - 5*0 + C \qquad = 0 + C$$

So

$$| s(3) - s(0) | = \left| \frac{-3}{2} + C - (0 + C) \right|, \qquad \textit{notice the C's cancel.}$$

$$= \frac{3}{2} = 1.5$$

The displacement of the particle is 1.5 meters.

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Finding the Distance Traveled:

Notice that v(0) = 3*0 – 5 = –5 and v(3) = 3*3 – 5 = 4.

Since the velocity flips from negative to positive we know the particle started out moving "left" and ended moving "right." So it obviously did NOT travel in the shortest distance. We must find out how far it traveled "left" and how far it traveled "right." To do this we need to know where it switched directions on the interval [0, 3]. Notice to switch directions the velocity must be zero at some time. To find out when we set v(t) = 0 and solve for t:

$3*t - 5 = 0$

$3t = 5$

$t = 5/3$

So from t = 0 to 5/3 the particle was moving left. The distance it traveled in that time = | s(5/3) – s(0) |. From above we know $s(t) = (3/2)*t^2 - 5t + C$.

$$s(5/3) = \frac{3}{2}*\left(\frac{5}{3}\right)^2 - 5*\left(\frac{5}{3}\right) + C \qquad = \frac{-25}{6} + C$$

$$s(0) = \frac{3}{2}*0^2 - 5*0 + C \qquad = 0 + C$$

Distance traveled left = | s(5/3) – s(0) |

$$= \left|\frac{-25}{6} + C - (0 + C)\right|, \qquad \textit{the C's cancel}$$

$$= \frac{25}{6} \text{ meters.}$$

From t = 5/3 to 3 the particle traveled right. This distance = | s(3) – s(5/3) |.

$$s(3) = \frac{3}{2}*3^2 - 5*3 + C \qquad = \frac{-3}{2} + C$$

$$s(5/3) = \frac{3}{2}*\left(\frac{5}{3}\right)^2 - 5*\left(\frac{5}{3}\right) + C \qquad = \frac{-25}{6} + C$$

Distance traveled right = | s(3) – s(5/3) |

$$= \left|\left(\frac{-3}{2} + C\right) - \left(\frac{-25}{6} + C\right)\right|, \textit{ the C's cancel}$$

$$= \frac{8}{3} \text{ meters}$$

The total distance traveled = $\frac{25}{6} + \frac{8}{3} = \frac{41}{6} \cong$ **6.833 meters**.

80. The acceleration function in meters/second^2 and the initial velocity are given for a particle moving along a curve. Find (a) the velocity at time *t* and (b) the distance traveled during the given time interval.

$a(t) = 2t + 3, \quad v(0) = -4, \quad 0 \le t \le 3$

Finding the velocity function:

Recall that acceleration is the derivative of velocity, thus the integral (antiderivative) of acceleration is velocity. The trick is finding the +C at the end. To do that we will use the given $v(0) = -4$.

$v(t) = \int (2t + 3)\, dt \quad = t^2 + 3t + C$

Now we find out what C is:

$v(0) = 0^2 + 3*0 + C$, *and we are given v(0) = -4.*

$-4 = 0 + 0 + C$

$C = -4$

$v(t) = t^2 + 3t - 4$

Finding the distance traveled:

This is NOT the same thing as finding the displacement of the particle.

Displacement is how far the particle is from where it started moving.
Distance traveled is how far the particle moved to get to where it ends.

Basically what this means is the particle does NOT have to move in a straight line. For example to get to Dallas from College Station, you might go through Waco or you might go to Houston and then to Dallas. Obviously the distance traveled would NOT be the same, but your displacement would be.

Recall that velocity is the derivative of position, $v(t) = s'(t)$. Thus the integral (antiderivative) of velocity is position. So the displacement could easily be found by taking the absolute value of (position at start time) – (end position), or rather | s(start time) – s(end time) |. Finding the distance traveled is slightly more difficult.

To find the distance traveled we need to find out if and when the velocity changes direction (like the particle was moving west, negative direction, and then switched to moving east, the positive direction).

In our case we see that the $v(0) = -4$, but $v(3) = 9 + 9 - 4 = 14$.

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As the sign on the velocity changed we know that the particle must have changed direction sometime. You can also look at the graph of v(t) and see that it is positive and negative on [0, 3]. So we need to find out where it switched, specifically we need to find out when v(t) = 0. So we set v(t) = 0 and solve for t:

$$t^2 + 3t - 4 = 0$$
$$(t + 4)(t - 1) = 0$$

So velocity is 0 at t = -3 and t = 1. As we are looking at the interval [0, 3] we only need to worry about t = 1.

In sum we know that from t = 0 to 1 the particle was moving "left" because the v(t) is negative from t = 0 to 1. We also know the particle was moving "right" from t = 1 to 3 because v(t) is positive from t = 1 to 3.

To find the distance traveled ("left") from t = 0 to 1 we need to calculate:

$$\begin{aligned} |s(1) - s(0)| &= \left|\int_0^1 (t^2 + 3t - 4)dt\right| \\ &= \left|\left[\frac{t^3}{3} + \frac{3t^2}{2} - 4t\right]_0^1\right| \\ &= \left|\left(\frac{1}{3} + \frac{3}{2} - 4\right) - (0 + 0 - 0)\right| \\ &= \left|\frac{-13}{6}\right| \\ &= \frac{13}{6} \end{aligned}$$

Thus the particle moved $\frac{13}{6}$ meters to the left.

Now we find the distance the particle traveled ("right") from t = 1 to 3:

$$\begin{aligned} |s(3) - s(1)| &= \left|\int_1^3 (t^2 + 3t - 4)dt\right| \\ &= \left|\left[\frac{t^3}{3} + \frac{3t^2}{2} - 4t\right]_1^3\right| \\ &= \left|\left(\frac{27}{3} + \frac{27}{2} - 12\right) - \left(\frac{1}{3} + \frac{3}{2} - 4\right)\right| \\ &= \left|\frac{38}{3}\right| \\ &= \frac{38}{3} \text{ meters to the right} \end{aligned}$$

The total distance traveled is thus $\frac{13}{6} + \frac{38}{3} = \frac{89}{6} \cong$ **14.833 meters**.