Section 6.5
Solutions and Hints

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for the book:
<u>Calculus, Early Vectors</u>
by James Stewart.

The substitution rule takes practice.
You usually will know to use it under three conditions:

a. You have (something)*(other thing)$^{\text{some power}}$ and (something) looks a lot like the derivative of (other thing).
   Notice that (some power) could be a negative power or fractional power – like square root.
b. You have (something)*sin(other thing) or (something)*cos(other thing) or (something)*tan(other thing) and the (other thing) is a constant times a variable (like 4θ) or (other thing) is a variable to a power (like t^3).
c. Nothing else works

In all these cases you will most often let u = (other thing), find du, solve for dx and substitute everything into the original equation. You will have done it correctly if things cancel out nicely and you get an integral you can solve. =)

8. Evaluate the indefinite integral: $\int x^3 * (1- x^4)^5\, dx$

Here we have (something) = x^3 and (other thing) = $(1 - x^4)$.
Notice the derivative of $(1 - x^4) = 4*x^3$. So set up the following:

$$u = 1 - x^4$$

$$du = 4*x^3dx \quad \rightarrow dx = \frac{du}{4x^3}$$

Thus

$$\int x^3 * (1-x^4)^5\, dx = \int x^3 * (u)^5\, dx, \qquad \textit{Put u in for } 1 - x^4.$$

$$= \int x^3 * (u)^5 \frac{du}{4x^3}, \qquad \textit{Put } \frac{du}{4x^3} \textit{ in for dx.}$$

$$= \int \frac{u^5}{4}\, du, \qquad \textit{Simplifiy.}$$

$$= \frac{u^6}{24} + C$$

$$= \frac{(1-x^4)^6}{24} + C, \qquad \textit{Put } 1 - x^4 \textit{ in for u.}$$

$$\int x^3 * (1-x^4)^5\, dx = \frac{(1-x^4)^6}{24} + C$$

19. Evaluate the indefinite integral.

$$\int \frac{x}{\sqrt[4]{x+2}}\,dx$$

Here our (something) = x and (other thing) = (x + 2) with (some power) = -1/4.

Notice the derivative of (x + 2) = 1*dx. So initially it would appear substitution won't work. However if we let u = x + 2 then solve for x we get x = u – 2 and nice things will happen:

u = x + 2 → u – 2 = x

→ du = dx

Now we put stuff in:

$$\int \frac{x}{\sqrt[4]{x+2}}\,dx = \int \frac{u-2}{\sqrt[4]{u}}\,du, \qquad \textit{and if we simplify we get:}$$

$$= \int \frac{u-2}{\sqrt[4]{u}}\,du$$

$$= \int \frac{u}{\sqrt[4]{u}} - \frac{2}{\sqrt[4]{u}}\,du$$

$$= \int u^{3/4} - 2*u^{-1/4}\,du$$

$$= \frac{4u^{7/4}}{7} - 2*\frac{4u^{3/4}}{3} + C, \qquad \textit{note: } u^{7/4} = u*u^{3/4}.$$

$$= \frac{4*u*u^{3/4}}{7} - \frac{8*u^{3/4}}{3} + C, \quad \textit{find a common denominator.}$$

$$= \frac{3*4*u*u^{3/4}}{21} - \frac{7*8*u^{3/4}}{21} + C$$

$$= \frac{12*u*u^{3/4} - 56*u^{3/4}}{21} + C, \qquad \textit{take a } u^{3/4} \textit{ “out”.}$$

$$= \frac{u^{3/4}*(12u-56)}{21} + C, \qquad \textit{put x + 2 in for u.}$$

$$= \frac{(x+2)^{3/4}*(12*(x+2)-56)}{21} + C, \quad \textit{simplify.}$$

$$= \frac{(x+2)^{3/4}*(12x+24-56)}{21} + C$$

$$= \frac{(x+2)^{3/4}*(12x-32)}{21} + C$$

$$\int \frac{x}{\sqrt[4]{x+2}}\,dx = \frac{(x+2)^{3/4}*(12x-32)}{21} + C$$